Addendum on Alkhwarizmi: A Table Found?

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Alkhwarizmi's "original" astronomical rules (provisionally defined as independent of Maslama's revision, ed. Suter 1914) are preserved in two main sets of fragments, namely, in Ibn Almuthanna's commentary (Hebrew and Latin versions, ed. Goldstein 1967 and Millás 1963), and in some Latin canons for the Toledan Tables, including the rules edited in CIMAGL 62 (31-75, *q.v.* for the refs.). In a section on finding area digits of solar eclipses (Goldstein's Q82 p.137:20; my Oo171) both these sources refer to a "table of magnitude", with values in digits and minutes, unknown from Maslama. The table itself is absent in both cases; Goldstein has a reconstruction p.240.

A copy of such a table survives in ms. Cambridge, Trinity Coll., O.8.34, 31v (early 13th c.), unexpectedly within a set of normal "Toledan" eclipse tables, as follows:

Numerus		Tabula magnitudinis secundum Alchoarizmi de sole.		
Puncti	Minuta	Puncti	Minuta	Secunda
0	30	0	10	15
1	0	0	28	43
1	30	0	52	5
2	0	1	18	48
2	30	1	48	18 *
3	0	2	20	48
3	30	2	54	53
4	0	3	30	8
4	30	4	6	37
5	0	4	43	17 *
5	30	5	21	50
6	0	6	0	0

This essentially confirms Goldstein's reconstruction. For a summary check of the extra values, note that the table is meant to render the area of a segment of a circle, given its sagitta x. The radius is assumed to be 6, and the total area of the circle is expressed as 12 area digits. Then the half-chord $y = (x(12-x))^{\frac{1}{2}}$, and the area digits of the segment can be found as, e.g.,

$$a = \{36 \, \mathrm{ArcTan}(y/(6-x)) - y(6-x)\} / 3\pi, \quad 0 \le x < 6,$$

calculating in the modern way for convenience. Even then, the fit is fairly good: one finds discrepancies of 12" or less between our table and the calculation except at 2^P;30 (40" calculated; XVIII *ms.*, perhaps from X'VIII = 48), and at 5^P;0 (57" calculated; XVII *ms.*, perhaps an Arabic "57" mis-transcribed, cf. Toomer 1968, p.28).

It is true that the sources do not mention the second-column of our table; and it remains to be seen whether any special sine-values, such as the Indian kardaga-values, were used in computing it. Still the present observation confirms the likelihood that the older collections of Toledan tables, just like the canons meant for them, may have received a lot of Alkhwarizmian matter independently of the Maslama tradition.