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Enuntiabilia, Consequences, and Impossible Antecedents. Some New Evidence from ms. Paris BNF, lat. 3713, fols 86r–89v

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Introduction: Old questions, a new text. The *Tractatus Parisiensis de enuntiabilibus*

The codex Latin 3713, preserved in the French National Library and originally part of the Abbey of Saint Martial's library in Limoges, is a composite manuscript including texts of different content, which are datable to the twelfth and thirteenth centuries. Among these texts, we find two short logical treatises, included in fols 86r–88v and 88v–89v respectively, written by two different hands around the third or fourth quarter of the twelfth century.[1] The codex's modern list of contents, contained in the second front guard, registers the two treatises together under the label "*fragmentum dialecticae.*" These two texts were first

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[1] The manuscript is available at: https://gallica.bnf.fr/ark:/12148/btv1b9078154m. I am grateful to Sofia Orsino for suggesting this dating to me in private conversation.

discovered by Yukio Iwakuma, who listed them among the sources connected to the school of the *Parvipontani*, but their content has received no further attention in the secondary literature so far.[1] Like many other logical sources from this period, the texts in question are anonymous; the only clue that we have to the context of production of the second treatise is a brief comment, at the end of fol. 88v, saying that the phrase "you [are/were?] in England" is an example of a *per accidens* necessary statement, namely, something that could have been false in the past but is currently true. Some logical and ontological views offered in the text are compatible with what we know about the teachings of the *Parvipontani*, especially for what concerns the general theory of assertables and the idea that everything follows from an impossible antecedent. Nevertheless, because recent studies are significantly reshaping our knowledge of the logical schools in the mid-twelfth century, including their discussion of (impossible) *enuntiabilia*, I prefer to refrain here from attributing the text to a specific school, and I will thus refer to the two logical treatises as *Tractatus Parisiensis de enuntiabilibus I* and *II* (henceforth, *TEn I* and *Ten II*).

Both texts contain remarks on the nature and logic of assertables (*enuntiabilia*), that is to say, items that are suited to be asserted or said by means of oral and written statements. These are indicated in Latin with accusative and infinitive clauses.[2] In the first of the two treatises, the

[1] Iwakuma proposes these titles in his personal description of the manuscript, which he has made accessible via the following folder: https://drive.google.com/drive/folders/1HRY2FckntQ7lJfx1FTFg99aF1sByLwRu?usp=sharing.

[2] The English translation of Latin expressions of this kind poses problems. As pointed out by Martin (2022a, p. 112), the standard way of translating them with *that*-clauses may be problematic, because there is a difference in the logic of this period between accusative and infinitive clauses and *quod*-clauses. For this reason, in this article I follow Martin and leave these Latin expressions untranslated, in an effort to prioritize precision over readability. In *TEn I*, assertables are sometimes replaced with the corresponding sentences, which I translate into English—this occurs especially within demonstrations, although not consistently. The assertable *Socratem esse asinum* corresponds to the sentence "Socrates is a donkey" (*Socrates est asinus*).

content is structured in a rather systematic way, apparently with a didactic intent. The second treatise seems to be a shorter version, like a summary or a compendium, of the first. Many of the questions that are raised in the two texts echo similar debates taking place among the logical schools in Paris in the second half of the twelfth century, especially in relation to the question of what assertables are, whether their existence is eternal and independent of the existence of the things they are about, how they can be divided into categories or kinds, and how assertables falling into these categories may be structured to compose valid consequences or true conditionals. As Iwakuma and others have pointed out extensively,[1] the notion of *enuntiabile*, or *dictum*, underwent a philosophical revival in the middle and the end of the twelfth century, and was widely discussed by the school of the *Parvipontani* (whose master, Adam of Balsham, seems indeed to have coined the term "*enuntiabile*"), as well as by the *Meludinenses*, the *Porretani*, the *Nominales* and the *Albricani*. By the end of the century and especially after the gain of popularity of Peter Lombard's *Sentences*, the reflection on assertables also stimulated questions of doctrinal and theological content, concerning their eternal nature in relation to God's own eternity and the immutability of his knowledge.[2]

TEn I and *II* do not have much to say on the metaphysics of *enuntiabilia*. Their author explicitly endorses the eternity of all assertables and seems to confer an ontological status on both true and false *enuntiabilia*, which are said to be "something" (*aliquid*) and which he categorises into a complex taxonomy of "kinds" (*genera*). However, these views are simply assumed, almost without argumentation, and without delving into their metaphysical or theological consequences. There are some attempts in the texts to reject the views of others on assertables (such as the idea that nothing follows from a falsehood, or that false assertables

[1] See Nuchelmans 1973, p. 169; Iwakuma 1997, p. 19. On *enuntiabilia* in this period of the history of logic, see also Kneepkens 1997 and Wilks 2008, pp. 145–48. More recent studies on *enuntiabilia* in the teaching of these schools are found in Martin 2022a and Donato 2023.

[2] For the theological discussions on assertables, see Iwakuma 1997.

do not exist), but the reported conflicts concern the logic of *enuntiabilia* rather than their ontology. With respect to the question of what assertables are, then, the two Parisian treatises are not advancing any innovative or sophisticated theories, and seem to merely report the general opinion of a master or a school.

Nonetheless, the two texts are particularly noteworthy, I believe, for the detailed analysis they offer of *impossible assertables*. In both treatises, unusual attention is paid to the notion of impossibility: the author advances a distinction between *per se* and *per accidens* impossibility, adding that only assertables that are *per se* impossible are proper antecedents to every other assertable. A similar terminology may be found in other contemporary sources dealing with the *ex impossibili quodlibet* principle,[1] but nowhere is it as extensively discussed as in the two Parisian texts. Moreover, both *TEn I* and *TEn II* further subdivide *per se* impossibility into six sub-kinds, distinguishing impossible assertables such as *Hominem esse asinum* from the other sorts of impossibility expressed in, e.g., *Socratem esse Platonem* or *Socratem sedere et non sedere*. This subdivision is, to my knowledge, unique among parallel debates of this time. For each of these "kinds of impossibles" (*genera impossibilium*), the two Parisian texts aim to prove that all assertables follow from them (*ex omni impossibili per se—et eo solo—omne sequitur enuntiabile*, cf. fol. 86r). A study of the two logical treatises will thus provide, as I hope, new evidence for the interpretation of early medieval discussions of counterpossibles and of the logic of impossible statements. To this end, in what follows I shall highlight the passages in the texts that I find most relevant in this respect, and connect them with parallel discussions on the

[1] I mention here this version of the principle, namely that from an impossible *anything* follows (*ex impossibili sequitur quodlibet*), because it is the most known and used by contemporary interpreters of this medieval debate. However, *TEn I* uses a slightly different version, that is, that from an impossibility *everything*, or every assertable, follows (*ex impossibili omne sequitur enuntiabile*). In John of Salisbury's *Metalogicon*, we find a formulation similar to the latter one for a different, though closely related, logical principle, namely that from an impossible every other impossible follows: *ex uno impossibili omnia impossibilia provenire* (1991, II 10, 72).

same topic. In this contribution, I will confine my analysis to *Tractatus Parisiensis de enuntiabilibus I*, and only to those parts that discuss the general theory of assertables or the *ex impossibili quodlibet* principle.

Before proceeding, it may be helpful to offer an overview of the contents of *TEn I*, which may be divided into four main sections. The first includes the definition of assertables, some general remarks on their eternity and their role in the construction of arguments and hypothetical propositions, as well as their twofold division into kinds: true and false assertables on the one hand, possible and impossible assertables on the other (the division is presented in slightly different terms in *TEn II*). The treatise's second section focuses on impossible assertables, which, as was said, are further divided into several sub-categories. For each kind of *per se* impossible assertable, the author offers one or more proofs to show that everything follows from it. These proofs always apply the same inferential strategy, apparently aiming to trace back all cases to the one considered as the first and principal kind of impossibility, exemplified by the assertable *Hominem esse asinum*. After giving proofs in favour of the *ex impossibili* principle for all kinds of *per se* impossibility, in the third section the author proceeds to consider assertables that follow from every other assertable, which he identifies as the *per se* necessary ones. Finally, the fourth section of the treatise discusses inferential relations between assertables that are neither necessary nor impossible, with a focus on affirmative, *de praesenti* ones. As mentioned, my analysis will only focus on the first two sections, which amount to approximately the first half of the entire treatise. The Appendix to this article includes a critical edition of this portion of the text.

1 The nature and division of assertables

As was mentioned, *TEn I*'s approach to assertables is logical rather than metaphysical. In the opening lines of the treatise, the author explains that assertables—defined as what is signifiable or sayable by an assertion (*enuntiatio*)—are the constituents of any relation in which something is said to "follow from" or to be "proved on the basis of" something else. And since logic is a discipline that is mostly concerned with the proving

and disproving of things, the constituents of these relations of following must be investigated. In the first paragraphs of the text, the author also provides a definition of consequence, which he apparently applies to both arguments and conditionals. This definition is given using the standard modal account of following, enriched with a temporal element: an assertable follows from another one if, at all times, it would be impossible for the latter to be true and the former false; or, in other words, if the antecedent is, was, or will not be able to be true without its consequent (see §3 in the Appendix). Originating from Boethius' modal account of consequence in *De Hypotheticis Syllogismis*,[1] according to which a consequence is good if it is impossible for the antecedent to be true and the consequent false, this criterion was not unanimously adopted in twelfth-century logic, as is well known, but is still common in logical treatises dating to the late twelfth and early thirteenth centuries, in connection with the discussion of impossible antecedents.[2]

At the beginning of *TEn I*, every assertable is said to be eternal and to have a necessary existence, in the sense that each of them is uncreated and can never cease to be (see §1 in the Appendix). The author, however, does

[1] See e.g. Boethius 1969, 1.9.4–5: "Necessitas vero hypotheticae propositionis, et ratio earum propositionum ex quibus iunguntur inter se connexiones, consequentiam quaerit, ut cum dico: 'Si Socrates sedet, et vivit' neque sedere eum, neque vivere necesse est sed, si sedet, vivere necesse est. Item cum dicimus: 'Si sol movetur, necessario veniet ad occasum' tantumdem significat quantum, si sol movetur, veniet ad occasum. Necessitas enim propositionis in consequentiae immutabilitate consistit. Item cum dicimus: 'Si possibile est legi librum, possibile est ad versum tertium perveniri' rursus necessitas consequentiae conservata est; nam si possibile est legi librum, necesse est etiam id esse possibile, ut ad versum tertium perveniatur. Opponuntur autem hypotheticis propositionibus illae solae quae earum substantiam perimunt. Substantia vero propositionum hypotheticarum in eo est, ut earum consequentiae necessitas valeat permanere." See also Boethius 1969, 1.9.6: "Si quis igitur recte conditionali propositioni repugnabit, id efficiet ut earum destruat consequentiam, veluti cum ita dicimus: 'Si A est, B est' non in eo pugnabit si monstret, aut non esse A, aut non esse B, sed si posito quidem A, ostendit non statim consequi esse B sed posse esse A, etiamsi B terminus non sit."

[2] For a discussion of late twelfth- and early thirteenth-century sources applying this definition of consequence in connection with the *ex impossibili* principle, see Spruyt 1993 and Binini 2024.

not discuss further whether assertables have a *sui generis* ontological status, nor does he consider at any point the theological implications raised by assertables' sempiternal existence. As has been pointed out, the eternity of assertables, and thus their independence from the existence of the *res* they are about, was endorsed not only by the *Parvipontani* but also, it seems, by the schools of the *Albricani* and *Meludinenses*, even if the latter restricted this claim only to some true assertables.[1] Other schools, like that of the *Porretani*, rather conceived assertables as the composition or division of things, and thus claimed that their existence depends on the beginning and ceasing of their constituents.[2]

As was quite common in twelfth-century discussions of this topic, assertables are also said to be the bearers of truth and falsity in *TEn I*. The treatise considers the two truth values in terms of "kinds" (*genera*) assertables fall into, and the author specifies that no assertable may fall outside these two kinds. Moreover, it is claimed that no other item, apart from assertables, can be properly said to be true or false (§2 in the Appendix). Interestingly, in the same passage the author also treats assertables as the exclusive bearers of modal properties, being either possible, impossible, or necessary.[3] This is in line with the *de dicto* interpretation of modal claims that we may see throughout the entire treatise.[4] Although the author considers all assertables as immutable with respect to their existence, still he seems to believe that assertables may

[1] See Iwakuma 1997; Martin 2022a.

[2] For a recent analysis of the *Porretani*'s views on assertables, see Donato 2023.

[3] Contingency is never counted among modalities in either *TEn I or TEn II*. This is interesting, because it may be a sign that this logical text was composed quite early, before the influence of Aristotle's *Prior Analytics*. Other sources from the second half of the twelfth century, like the *Ars Meliduna*, divide true *enuntiabilia* into necessary and contingent ones, using the term "contingent" not as synonymous with "possible" (like Abelard and others did at the beginning of the century) but in opposition to necessity (see footnote 2 on p. 339 below).

[4] See for instance §11 in the Appendix, where the author considers the opinion of some according to whom it would be true to state, for instance, that it is possible for a white thing to be a black thing. This reading (which gains sense only within a *de re* reading of modals) is criticised in *TEn I*.

vary with respect to the category they belong to: they may pass from being true to being false and vice versa, and unidirectionally from being possible to being impossible, or from non-necessary to necessary.

On the basis of the two main divisions of assertables into true/false and possible/impossible, the author of *TEn I* then continues his taxonomy by identifying two kinds of true assertables (necessary and non-necessary) and two kinds of false ones (possible and impossible). Necessary and impossible assertables are further divided into a *per se* and *per accidens* category, and *per se* impossibles into six sub-categories, as will be detailed in Section 2 below. Modal notions are not themselves defined on the basis of more primitive concepts, but are simply analysed in terms of the (im)possibility of being true at some/all/no moment of time. Necessity, on the other hand, is defined as the impossibility of being otherwise, in a characterisation for which the author of the *Tractatus* invokes Boethius as authority, and that is often used in early twelfth-century works on modalities.[1] A slightly different division of assertables is proposed in *TEn II*: here six categories are identified, namely (i) true necessary *per se*; (ii) true necessary *per accidens*; (iii) non-necessary true; (iv) false possible; (v) false impossible *per accidens*; (vi) false impossible *per se* (see fol. 88v). Similar categorisations of *enuntiabilia* based on truth and modal values are advanced in contemporary debates, as for instance in the *Ars Meliduna*.[2]

[1] See for instance Peter Abelard's *Dialectica*: "*Necessarium*" *autem id dicit quod ita sit et aliter esse non possit*. (1970, p. 194; see also p. 272). The same definition is used in other early twelfth-century treatises on modalities, such as the commentaries on *De Interpretatione* labelled *H9* and *H11* in Marenbon's catalogue (see Marenbon 2000) and preserved in mss. Orléans, Bibliothèque municipale, 266, pp. 5a–43a; Assisi, Biblioteca Conventuale Francescana, 573, fols 48rb–67vb; Paris, Bibliothèque Nationale de France, lat. 13368, fols 225r–31r; Vatican City, Biblioteca Apostolica Vaticana, reg. lat. 230, fols 80r–87r.

[2] The *Ars* is preserved in ms. Oxford, Bodleian Library, Digby 174, ff. 211ra–241rb. An analysis and some excerpts of it are edited in De Rijk 1967, 264–390 and in Iwakuma 1993. For the passage in question, see *Ars Meliduna* IV, fol. 237vb: "Horum [i.e. enuntiabilium verorum] alia sunt necessaria, alia contingentia. Necessaria sic a plerisque

A few general inferential rules are enumerated that govern the logical relations among these kinds of assertable: from true assertables only other true assertables may be derived; similarly, from possible ones, only possible assertables follow. The latter principle was well known already at the beginning of the twelfth century, and is mentioned for instance by Abelard in the *Dialectica* (1970, p. 202). More controversial in the logic of the period were inferences drawn from false and impossible premises: according to the author(s) of *Tractatus Parisiensis I* and *II*, from what is false, both true and false assertables follow. Explicit mention is made in *TEn I* of the opinion of some *perversi disputatores* who deny that the false could be antecedent or consequent in a true conditional (cf. §6 in the Appendix): these "depraved dialecticians" may be easily identified with the *Meludinenses*, who defended the view that *nihil sequitur ex falso*. Of impossible assertables, *Tractatus Parisiensis* claims that they can be antecedent to both possible and impossible conclusions. Only impossibilities that are *per se*, however, entail all other assertables. To impossible assertables and their logic I dedicate the next two sections.

2 The taxonomy of impossible assertables

Once he has mentioned the idea that from an impossible assertable everything follows, the author points out that this principle should be restricted to a specific kind of impossibility, that is, *per se* impossibility, as opposed to impossibility *per accidens*. A similar position is endorsed in *TEn II*.[1] As the author characterises it, the distinction between *per se* and *per accidens* depends on whether something is impossible at all times (e.g.,

describitur 'necessarium est enuntiabile necessario verum'; ab aliis vero 'verum quod non potest esse falsum'. Impossibile quoque dupliciter. Similiter verum contingens 'verum quod potest esse falsum' aut 'verum quod non necessario est verum'."

[1] On the basis of the aforementioned division of assertables into six kinds, the sixth of which applies to *per se* impossibilities, the author remarks that assertables of any other kind follow from those belonging to the sixth kind, whereas *per se* impossibilities never follow from assertables within other categories, see fol. 89r: "Ex sexto [genere] sequitur sextum et quidlibet superiorum, et ipsum ex nullo eorum … Item generaliter est sciendum quod ex omni enuntiabili sexti generis sequitur omne enuntiabile cuiuslibet generis."

the fact that Socrates is a donkey), or instead that it is impossible now and from the present moment onwards, but could have been true in the past (e.g., that I did not exist). This distinction is not unique to *TEn I* and *II*. In fact, it is rehearsed in a few other sources from the second half of the twelfth century,[1] and it will again be used in later logical debates, often in connection with discussions of the *ex impossibili quodlibet* principle. For example, we find it in the *Ars Meliduna*, whose author divides necessary assertables into *per se* and *per accidens* ones, and then mentions that the impossible is similarly twofold.[2] A reference to *per se* impossibility is also found in Alexander Neckham's *De naturis rerum* (see 1863, pp. 288–89), in the fragment preserved in ms. Leipzig, Universitätsbibliothek, Fragm. lat. 32, ff. 1–3, and in the logical treatise contained in ms. Paris, Bibliothèque Nationale de France, lat. 2904, p. IVb.

While the terminology of *per se/per accidens* impossibility will be used by later logicians in several different meanings, in the period of the twelfth-century logical schools the distinction is made primarily in temporal terms, as it is in *TEn I*. By the end of the century and the beginning of the next one, however, the division between the two kinds of impossibility will also be characterised in different ways, for instance as a distinction between "metaphysical" impossibilities, expressing a natural repugnance or incompatibility between things (e.g., "A human is a donkey"), as opposed to things that are impossible only "with respect to a certain determination" (e.g., "Socrates is white in this instant").[3] In other cases, the same distinction is used to discriminate between natural and

[1] To my knowledge, however, in no logical text earlier than the mid of the twelfth century do we find a distinction of this sort applied to impossibility, even though a similar division between absolute and temporally qualified modalities is applied already in the logic of the early 12th century. On this, see Binini 2021, pp. 33–44.

[2] See *Ars Meliduna* IV, fol. 237vb: "Item necessariorum aliud necessarium per se, aliud per accidens. Necessarium per se quod nullo casu fit necessarium, sed semper fuit et est et erit necessarium, ut *deum esse.* Necessarium per accidens quod accidentaliter, id est aliquo rerum eventu, incepit esse necessarium, ut *Cesarem fuisse.* Impossibile quoque dupliciter."

[3] See for instance the use of this distinction in the early thirteenth-century *Tractatus Emmeranus de positione falsa*, ed. in De Rijk 1974, p. 113.

supernatural impossibilities—a virgin giving birth being an example of something that is naturally impossible but not *per se* impossible, since it could be brought about by God with his supernatural power.[1] The temporal characterisation of the distinction between different kinds of impossibility, however, continues to be prevalent by the end of the twelfth century and will still be used by many thirteenth-century philosophers and logicians, such as William of Sherwood, Roger Bacon, Nicholas of Paris, or Thomas Aquinas.[2]

As already mentioned, in *TEn I* and *II* the claim is endorsed that only *per se* impossibilities are proper antecedents to any other assertable. See for instance the following passage from fol. 86r:

> From a false impossible [antecedent], both possible and impossible [consequents follow]. But in order for this principle to become clearer, you should consider that there are two kinds of true and false [assertables]: the true ones that are necessary *per se*, and the false ones that are impossible *per se*. From that which is impossible *per se*—and only from this—everything follows. Every assertable that is *per se* necessary—and only these—are entailed by every other assertable. (for the Latin text see §4 in the Appendix)

Indeed, we may suppose that the distinction between *per* se and *per accidens* impossibility was first formulated in the second half of the twelfth century with the aim of proposing a more fine-grained version of the *ex impossibili sequitur quodlibet* principle, rephrasing it as: "from what is impossible *per se* everything else follows."[3] We may also suppose that this version of the principle was indeed the one endorsed by the *Parvipontani*, or at least this is the version of the principle reported in the *Ars Meliduna* and in ms. Paris, Bibliothèque Nationale de France, lat. 2904

[1] See Stephen Langton, *Quaestiones theologiae*, I, q. 19a, par. 1, ed. Quinto and Bieniak, 402. On the discussion of *per se* impossibility in Langton's *Theological Questions*, see also Binini 2024 and Wciórka 2024.

[2] For the development of this distinction in the thirteenth and fourteenth centuries, see Binini 2022.

[3] For a more detailed defence of this claim, see Binini 2024.

as the opinion of some contemporary dialecticians, and also the version referred to and demonstrated by Neckham in his *De naturis rerum*.

What seems unique to *Tractatus Parisiensis de enuntiabilibus* is that, apart from dividing *per se* from *per accidens* impossibilities, the author further distinguishes *per se* impossible assertables into six distinct kinds, and for any assertable of these kinds he feels compelled to demonstrate that any other assertable follows from it. The first kind of impossibility identified, which is also said to be the primary kind (*unum et principale per se impossibilium genus*), is metaphysically defined and applies to assertables affirming of a certain natural substance that it is of a different species than the one it actually belongs to, as when we say that a human is a donkey. As is well known, this is the paradigmatic example of an impossible assertable in the second half of the twelfth century, and will continue to be so in thirteenth-century literature of *sophistaria* and *obligationes*, when the logic of impossible antecedents or premises is discussed. Two other kinds of impossibles listed may also be characterised as metaphysical impossibilities: they include assertables stating that a certain subject belongs to two incompatible natural species (as in *Socratem esse et hominem et equum*, the second kind), and assertables expressing the identity of two different individuals (*Socratem esse Platonem*, the third kind). There are then *per se* impossible assertables that we would characterise as "analytic" impossibilities, such as the union of two contrary terms (*album esse nigrum*, the fifth kind) and the affirmation and negation of the same property or predicate (*id quod est album non esse album*, *id quod sedet non sedere*, the sixth kind). The fourth kind of impossible assertable is the one that we would now consider as a proper logical contradiction or formal impossibility, namely, the combination of two contradictory statements into one single assertable, as in *Socratem esse; Socratem non esse*. It may be noteworthy that, in the Parisian text, this is not expressed through a conjunction of contradictory opposites, in the form "P *and* not-P," but rather as a juxtaposition of contradictory assertables (see §10 in the Appendix). Also noteworthy is the fact that, while several other twelfth-century discussions of impossible antecedents are centred around the discussion of natural or metaphysical impossibilities (see e.g. the ones

included in ms. Avranches, Bibliothèque Municipale, 244 and in the *Ars Meliduna*[1]), we have at least one other source, apart from *TEn I*, in which the demonstration of the *ex impossibili quodlibet* principle starts from a formal or syntactic impossibility: Alexander Neckham's famous demonstration contained in *De naturis rerum*.

As briefly mentioned above, in both *TEn I* and *TEn II* the discussion of *per se* impossible antecedents parallels a discussion of *per se* necessary consequents, which are said to follow from every assertable.[2] *Per se* necessities are those that are true at all moments of time, and the examples given are the following assertables: *Quidlibet esse*; *Deum esse*; *Aliquid esse verum*; *Aliquid esse falsum*; *Omne verum esse verum*. These are all eternally true, the author says, and so no other assertable may be true or might have been true without them. Therefore, assertables of this sort follow from any antecedent (*Sic ergo ex quolibet enuntiabili quodlibet istorum sequitur*), just as *per se* impossibilities are antecedent to everything.[3] Some proofs are offered in favour of this principle: these may

[1] The passages on impossible antecedents included in these two sources have been edited in Iwakuma 1993. Recent analyses of the two texts in connection to false and impossible assertables and their logic may be found in Lenzen 2021; Martin 2022a and 2022b; Binini 2024.

[2] We find mention of this principle also in other sources from the second half of the twelfth century, such as the *Ars Meliduna*, where the principle is attributed to the *Parvipontani* (see *Ars Meliduna* III, fol. 236ra: "Contra id vero quod praediximus ad categoricam non sequi continuativam, sic obiciet Parvipontanus sustinens necessarium ex quolibet enuntiabili sequi: 'si Socrates est homo, Socrates est animal, ergo si est homo, est animal si est homo' "); cf. Iwakuma 1993, p. 138; Ebbesen and Iwakuma 1992, p. 178.

[3] Since this part of the text is not included in the partial edition given in the Appendix, I report here the *incipit* of the discussion of necessary consequents contained in *TEn I* (fol. 87r): "Dicto ex quo enuntiabilium genere omne sequitur enuntiabile, nunc dicere restat, quod genus enuntiabilium sequatur ex omni enuntiabili. Hoc autem est necessarium per se, iuxta hanc regulam: omnis conditionalis hypothetica in qua sequitur necessarium per se, vera est, quemadmodum omnis hypothetica conditionalis est vera in qua antecedit impossibile per se. Sunt enim duo principalia enuntiabilium <genera>, secundum contradictionem ad invicem opposita: necessarium per se et impossibile per se. Ex quorum uno, omne sequitur enuntiabile, eorum vero reliquum sequitur ex omni

be the earliest demonstrations of the principle concerning necessary consequents in the history of logic, together with the one included in ms. Avranches, Bibliothèque Municipale, 244.[1]

enuntiabile, illud autem est impossibile per se, hoc vero necessarium per se, quod qualiter fiat ostendamus exemplis. Sunt autem necessaria per se huiusmodi: *Quidlibet esse*, *Aliquid esse*, *Aliquid esse verum*, *Omne verum esse verum*, *Omne falsum esse falsum*. Haec autem ex omni sequi enuntiabili sic ostendimus: Nullum enuntiabile potest vel potuit vel poterit (poterit] poterit vel *a.c.*) <esse> verum, nisi hoc esset verum: *Aliquid esse*; *Deum esse; Aliquid esse verum*, quod sic probatur: Quodlibet istorum ab aeterno fuit necessario verum, et est, et in aeternum erit necessario verum. Nec aliquid horum potest vel potuit vel poterit esse falsum. Ergo, nullum enuntiabile potest vel potuit vel poterit esse verum nisi quodlibet horum esset verum. Ergo, si aliquod enuntiabile est verum, quodlibet istorum est verum. Sic ergo ex quolibet enuntiabili quodlibet istorum sequitur. Amplius, si aliquod enuntiabile est, *Aliquod enuntiabile esse* est verum. Ergo, si aliquod enuntiabile est, *Nullum enuntiabile esse* est falsum. Ergo, si aliquod enuntiabile est, aliquid est verum et aliquid est falsum. Sic igitur haec duo, *Aliquid esse verum*, *Aliquid esse falsum*, ex omni enuntiabili sequuntur. Amplius, si aliquid est, *Aliquid* (aliquid] aliquod *a.c.*) *esse* est verum, et si aliquid est, *Aliquid esse vel non esse* est verum. Et si aliquid est, *unum esse* est verum. Ergo, si aliquid est, tria sunt quorum quodlibet est verum. Ergo, si aliquid est, trium quodlibet est. Et si quid est aliquid, ipsum est. Ergo, si aliquid (aliquid] aliquod *a.c.*) est, quidlibet est."

[1] See Iwakuma 1993, pp. 136–37: "Ex hoc facile est habere quod necessarium sequitur ad quidlibet; quia ex quo necessarii oppositum est impossibile, ex opposito sequitur quidlibet; sed ubi ex opposito necessarii sequitur aliquid, ex opposito illius consequentis sequitur oppositum illius antecedentis, scilicet necessarium; quia ex quo ex illo opposito (quod est impossibile) sequitur quidlibet, necessarium (quod illi impossibili opponitur) sequitur ad quidlibet. Verbi gratia secundum praedictam tenet *si Socrates est asinus, Socrates est capra*; ergo *si Socrates non est capra, Socrates non est asinus*, quia si sequatur aliquid ex alia <ex> opposito antecedentis sequitur oppositum consequentis, quare sequitur *si Socrates non est capra, Socrates non est asinus*. eadem modo *si Socrates [non] est capra, Socrates non est asinus*; quia sic sequitur *si Socrates est asinus, Socrates non est capra*, quare *si Socrates est capra, Socrates non est asinus*. et ita *Socratem non esse asinum*, quod est necessarium, sequitur ex utroque '*Socratem esse capram*' '*Socratem non esse capram*'. eodem modo potest haberi quod ex quolibet alio, et ita illud necessarium ex quolibet. eadem modo de quolibet alio necessario. Unde patet quod necessarium sequitur ex quolibet et impossibile antecedit ad quidlibet, sicut praehabitum est quod conclusio propter praedicta."

3 The logic of impossible antecedents and proofs that everything follows from them

Discussions of the principle according to which everything follows from the impossible are not uncommon in the second half of the twelfth century and at the start of the next century. We know about a few other sources datable to this period that offer proofs in favour of and against the principle, among them the *Ars Meliduna*,[1] the logical text contained in ms. Avranches 244,[2] the anonymous *Tractatus Vaticanus de communibus distinctionibus*[3] and Alexander Neckham's *De naturis rerum*.[4] In some of these sources, the principle concerning impossible antecedents is demonstrated by appeals to inferential strategies similar to the ones we find in *TEn I*, namely, by appealing to disjunctive addition and disjunctive syllogism, and sometimes the rules of simplification,[5] transitivity, and the

[1] The fragment of the *Ars Meliduna* that discusses the logic of impossible antecedents, which is found in ms. Oxford, Digby 174, ff. 236ra and 240va–b, has been edited in Iwakuma 1993, pp. 138–145. This part of the *Ars* has been recently commented upon in Lenzen 2021, which offers a reconstruction of the arguments and the connection between this text and the one included in ms. Avranches, Bibliothèque Municipale, 244. In Martin 2022a, Martin gives an analysis of the *Ars*' discussion of false (and impossible) *enuntiabilia* and the *Meludinenses*' idea that nothing follows from a falsehood.

[2] This text has been edited by Iwakuma in 1993, pp. 134–138. See Lenzen 2021 and Martin 2022b for recent discussions of its content.

[3] This treatise is found in ms. Vatican City, Biblioteca Apostolica Vaticana, lat. 7678, 73ra–82ra, and was edited in De Rijk 1988, 169–212.

[4] See Alexander Neckham 1863, pp. 288–89. There are also a few fragments in which a debate concerning the principle is reported, such as the ones included in ms. Leipzig, Universitätsbibliothek, Fragm. lat. 32, ff. 1–3 and in ms. Paris, Bibliothèque Nationale de France, Lat. 2904, p. IVb, the last of which shows similarities with the two Parisian treatises that are the subject of this article. I offer a survey of all known sources from this period dealing with the *ex impossibili quodlibet* principle in Binini 2024.

[5] It seems to me that in *TEn I* only predicate simplification is used, that is, the rule that allows one to infer, from an assertable with a conjunctive predicate, the assertable predicating one of the conjuncts, as in the inference from: *Socratem esse hominem et equum* to: *Socratem esse hominem*. The author never uses the propositional correlate of this rule, which grants the inference from e.g. "P and Q" to "P". The rule of addition (or disjunction introduction), on the other hand, is used propositionally in the treatise. On the

locus from opposites. What renders *TEn I* unique is that the same thesis—that from an impossible *per se* every other assertable follows—is proven for each of the six kinds of *per se* impossibility, and in all cases the author seems to replicate the same inferential pattern.

The first proof given in the text—which takes its cue from the first kind of *per se* impossibility, exemplified by the assertable *Socratem esse asinum*—is divided into two parts: first, the author proves that from the impossible antecedent corresponding to this assertable something true follows, namely that "If Socrates is a donkey, Socrates is a human." This first part of the proof is followed by an attack on the opinion of some "depraved dialecticians" who "deny that from the false something follows, or that the false follows from something else" (see §6 in the Appendix). A few arguments are then offered against this opinion. Granted thus that something does follow from the false, and specifically that "If Socrates is a donkey, Socrates is a human" is true, the author of *TEn I* goes on to prove that from the same antecedent everything else would also follow. Let us see this demonstration in its two steps. At first, some reasons are given, which the author calls "irrefutable," to prove that "If Socrates is a donkey, Socrates is a human." One of these reasons is the equivalence between the two assertables *Socratem esse hominem* and *Socratem esse aliquid*, which are said to be inseparable and derivable from each other, since at no moment of time can one be true and the other false—as the reader may recall, this was the definition of good consequence given in the *Tractatus*' opening section. Another reason appeals to the equivalence between *Socratem esse aliquid* and *Aliquid esse Socratem*, and claims that if Socrates is something, then this thing is either a human or a non-human; but the something that is Socrates is not something other than human; thus "If Socrates is something, Socrates is a human." If we accept these or the other reasons given for the claim that "If Socrates is something, then he is a human" (see §5 in the Appendix), it is easy to move, by transitivity, from "If Socrates is a donkey, Socrates is something" to "If Socrates is a donkey,

development of these inferential rules in the twelfth century in connection to the discussion of the *ex impossibili quodlibet* principle, see Martin 2022b.

Socrates is a human." Once this is granted, the author shows that from the impossible antecedent "Socrates is a donkey" everything follows. This second part of the proof may be reconstructed as follows:

1. If Socrates is a donkey, Socrates is a human;
2. If Socrates is a donkey, Socrates is a human or every assertable is true;
3. But if Socrates is a donkey, Socrates is not a human;
4. Thus, if Socrates is a donkey, every assertable is true.

This proof is similar to an argument offered in the aforementioned Avranches text, which also makes use of the rules of addition and disjunctive syllogism.[1] At the end of the proof, *TEn I* justifies the use of disjunctive syllogisms by saying that "when two [assertables] follow from the same [assertable] under disjunction, and if the contradictory of one disjunct also follows from the same antecedent, then the other disjunct follows from it."[2]

Similar inferential steps are at play in the other proofs advanced in *TEn I* in connection to the other kinds of *per se* impossibility. Take for instance the second among the listed *genera impossibilium*, namely assertables that predicate two incompatible species of a single substance, such as *Socratem esse et hominem et equum*. From impossibles of this kind too "we demonstrate that every assertable follows," in the following way:

1. If Socrates is a human and a horse, Socrates is a human;
2. If Socrates is a human, he is not a horse;
3. If Socrates is not a horse, Socrates is not a human and a horse;
4. If Socrates is a human and a horse, Socrates is not a human and a horse;
5. If Socrates is a human and a horse, Socrates is not a human and a horse or every other assertable is true;

[1] See Iwakuma 1993, pp. 135–36. For an analysis of this text and the proofs for the *ex impossibili* principle offered there, see Lenzen 2021.

[2] See §6 in the Appendix: "Ad huiusmodi argumentum confirmandum, datur haec regula: quotiens duo sequuntur ad idem sub disiunctione, si contradictio unius sequitur ad antecedens, reliquum sequitur ex eodem."

6. But, if Socrates is a human and a horse, it is not the case that (*falsum est quod*) Socrates is not a human and a horse;
7. Therefore, If Socrates is a human and a horse, every assertable is true.[1]

Noticeably, the author does not apply here another strategy he might have used, namely, the following one, picking up from step 3:

4*. If Socrates is a human and a horse, Socrates is not a human and a horse;
5*. If Socrates is a human and a horse, Socrates is a human and a horse;
6*. If Socrates is a human and a horse, Socrates is a human and a horse or every assertable is true;
7*. If Socrates is a human and a horse, every assertable is true.

One reason for not taking this path could be that the author wishes to avoid conditionals of the form "If P, then P," perhaps considering them as too "frivolous," or nugatory, to be included in the demonstration.[2] Another possible explanation is the author's intention to always maintain the same inferential structure in each demonstration of the *ex impossibili quodlibet* principle from every kind of *per se* impossibility. Indeed, in all the demonstrations given, the last three steps of the proof always have the form:

[1] I arrived at reconstructing this structure by supposing an emendation in the text, adding a *non* in the fifth step (see §8 in the Appendix).

[2] I am grateful to Wojciech Wciórka for suggesting this to me, and I am also grateful to an anonymous reviewer for their useful comments on this matter. In logical sources of this time, we find some references to *nugatio* in relation to the inference "If P, then P." For instance, while discussing the validity of the rules of conversion and the transposition of hypothetical statements, the *Ars Meliduna* reports the question of whether an inference in which the same assertion follows from itself is nugatory, an opinion that the author then rejects; see ms. Oxford, Bodleian Library, Digby 174, III, f. 235ra (I follow Iwakuma's unpublished transcription here): "Si dicamus consecutionem in qua idem ad se sequi ostenditur locutionem esse nugatoriam, ut 'si est homo, est homo, [est homo, est homo]', oportebit recipi consecutionem in transpositione; sed non videtur esse nugatoria, quia sicut categoricarum nulla magis est vera quam in qua idem de se, sic consequentiarum nulla magis est vera quam in qua idem ad se."

- *If P, then (R or Q);* where *Q* stands for any proposition whatsoever, or for the proposition "every assertable is true";
- *But if P, then, not R* (or: *it is false that R*);
- *Thus, if P, then Q*.

See, for instance, the similar proof given in relation to assertables such as *Socratem esse Platonem*. The author aims to demonstrate that "If Socrates is Plato, then Socrates is a horned goat," where the consequent is meant to stand for a random assertable:

1. If something is Socrates, that thing is not Plato;
2. If something is Plato, that thing is not Socrates;
3. Thus, if Socrates is Plato, Socrates is not Plato or Socrates is a horned goat;
4. But, if Socrates is Plato, it is false that Socrates is not Plato;
5. Therefore, if Socrates is Plato, Socrates is a horned goat.

Interestingly, the author of *TEn I* decides to adhere to this same demonstrative structure also when dealing with the fourth kind of impossibility, namely, assertables having the form of a syntactic contradiction, such as "*Socratem esse; Socratem non esse*" (see §10 in the Appendix). That every consequent follows from a contradiction of this sort may be proven quite easily thanks to propositional simplification, introduction of disjunction and disjunctive syllogism, in the way Alexander Neckham does in *De naturis rerum*.[1] The demonstrative path

[1] Cf. Neckham 1863, pp. 288–89 (translated in Martin 2018, p. 348): "I am amazed that some condemn the opinion of those saying that from what is per se impossible there follows any proposition (*enuntiabile*). This could be confirmed with many arguments or made clear with a few. For is it not the case that (1) if Socrates is a human being and Socrates is not a human being, then Socrates is a human being. But (2) if Socrates is a human being, then Socrates is a human being or a stone; therefore (3) if Socrates is a human being and Socrates is not a human being, then Socrates is a human being or a stone, but (4) if Socrates is a human being and Socrates is not a human being, then Socrates is not a human being; therefore (5) if Socrates is a human being and Socrates is not a human being, then Socrates is a stone."

chosen by the author of *TEn I*, however, is more convoluted than Neckham's, at least to our contemporary eyes, even though he seems to know and master all the inferential rules that Neckham later applies. This might be because, as was mentioned in the preceding section, the author of the *Tractatus* does not conceive impossible assertables of the form "*Socratem esse; Socratem non esse*" in terms of a conjunction of two propositions, but rather as a juxtaposition of two contradictory opposites, and this does not allow him to apply the rule of propositional simplification which Neckham applies in the first step of his proof. Another explanation might be that the author of *TEn I* conceives the rule of simplification only as predicate simplification—that is, applicable to the conjunction of two predicates, not of two propositions. Or again, his intention might be to repeat, also for this kind of *per se* impossibilities, the same inferential steps that he applies to impossibilities of all sorts, in order to prove the general replicability of his argumentative approach.

4 Conclusion: *Tractatus Parisiensis de enuntiabilibus* and William of Soissons' "machine"

I will not delve into the details of the last two demonstrations of the *ex impossibili quodlibet* principle listed in *TEn I*, which start from the fifth and sixth kind of *per se* impossibility, exemplified by the statements *album esse nigrum* and *id quod est album non esse album*. That everything follows from them is proven using the exact same inferential pattern already applied above. This repetition may appear redundant, but in fact the "mechanical" reiteration of the same demonstrative strategy is, in my opinion, one of the most interesting aspects of the *Tractatus*. As we have seen, the author aims to show that, from any given impossibility, we may always obtain the same paradoxical result by replicating the same demonstrative pattern consisting of a handful of logical moves. As the author intends to show, this result is drawn from *per se* impossible assertables of all kinds, including what we may call metaphysical, analytic, or logical impossibilities. This is different from what happens in the other sources of the time dealing with the *ex impossibili quodlibet* principle: the

proof offered in the Avranches text, for instance, only applies to "metaphysical" impossibilities such as *Hominem esse asinum*, while the proof offered by Neckham is restricted to contradictory antecedents having the form "P and not-P." Even though it is based on similar logical rules, the inferential scheme proposed in *Tractatus Parisiensis* has a generality that these other proofs lack. This makes this inferential scheme a very good candidate, I believe, for the identification of the famous logical "machine" invented by William of Soissons in order to "revolutionise the old logic" by "construing unthinkable consequences and destroying the theories of the ancients", and of which John of Salisbury speaks in the *Metalogicon* (II 10, 72).[1]

Although John does not explain in what exactly William's logical device (*machina*) consisted, we may suppose that it was a logical proof designed for deriving consequences from impossible antecedents, in a way that was not permitted in "the old logic," namely, in Aristotle's and Boethius' logic. On the one hand, as John suggests, William's logical device was used to contrast the Aristotelian principle according to which, from the same antecedent, a certain consequent and its own contradictory cannot both follow (that is, the principle claiming that, for any P and any Q, both Q and not-Q cannot simultaneously follow from P). This principle would be undermined, for instance, if we take an impossible statement as our P, and if we are able to demonstrate—as William's machine was perhaps supposed to demonstrate—that anything follows from an impossible. As has been already suggested by Martin,[2] William of Soissons' machine might thus have been the first known proof

[1] See John of Salisbury 1991, p. 72: "Interim Willelmum Suessionensem qui ad expugnandam ut aiunt sui logicae vetustatem, et consequentias inopinabiles construendas et antiquorum sententias diruendas machinam postmodum fecit, prima logices docui elementa, et tandem iam dicto praeceptori apposui. Ibi forte didicit idem esse ex contradictione cum Aristotiles obloquatur, quia idem cum sit et non sit, non necesse est idem esse. Et item cum aliquid sit, non necesse est idem esse, et non esse. Nihil enim ex contradictione evenit, et contradictionem impossibile est ex aliquo evenire. Unde nec amici machina impellente urgeri potui, ut credam ex uno impossibili omnia impossibilia provenire."

[2] To my knowledge, this suggestion was first made in Martin 1986.

demonstrating that from an impossible antecedent anything follows. This is certainly compatible with the words that John uses to describe William of Soissons' results, namely, as something both highly innovative and at the same time shocking or provoking paradoxical (*inopinabiles*) results. As is known, in mid- and late twelfth-century sources, proofs demonstrating that from an impossible statement all other statements can be derived are often accompanied by this paradoxical and provocative flavour. In addition, a few lines later, John of Salisbury mentions that William's argument was also supposed to demonstrate something that John himself says he is not willing to accept, namely, that "from one impossible antecedent all impossibles are drawn."[1] This further reinforces the idea that William's machine was aimed at proving that everything follows from an impossible antecedent, for indeed several twelfth-century proofs offered in favour of this principle demonstrate it by showing in parallel that from an impossible antecedent any other impossible statement follows: see for instance §8 and §9 in the Appendix below, where the author of *Tractatus Parisiensis de enuntiabilibus* establishes that everything follows from an impossible by showing that "If Socrates is both a human and a horse, then Socrates is both a cow and an archbishop," or that "If Socrates is Plato, then Socrates is a horned goat." These consequences, in which an impossible consequent is derived from an impossible antecedent despite their reciprocal irrelevance, are said, in the *Tractatus*, to be "similar" to the following claim: "if [impossible antecedent], then every assertable is true."

Although this is not said explicitly in John of Salisbury's text, there are thus good reasons to believe that the logical "machine" invented by William of Soissons was a formal proof demonstrating that from an impossible antecedent everything follows—likely the first such proof in the history of logic. This suggestion is reinforced by the connection acknowledged by John of Salisbury between William of Soissons and the teaching of Adam of Balsham: we now know that the school of the *Parvipontani* was the only school, in the middle and second half of the

[1] "Unde nec amici machina impellente urgeri potui, ut credam ex uno impossibili omnia impossibilia provenire."

twelfth century, endorsing the principle that from impossible antecedents anything can be derived. Because of the dating of the texts, their connection with the doctrines of the *Parvipontani*, the way in which the proofs are structured, and their similarity with some details contained in John of Salisbury's *Metalogicon*, I thus hypothesise that the *Tractatus Parisiensis de enuntiabilibus I* may be a representation of the powerful, innovative "machine" invented by William, a machine that, as Martin suggested, constituted a "turning point in the history of logic."[1] I hope that the partial analysis of *TEn I* offered here may help put together another piece of this history.

[1] Cf. Martin 1986, p. 565.

Appendix

Sigla
< > : *supplevi*
add.: *addidit*
corr.: correxi / correxit
a.c.: *ante correctionem*
p.c.: *post correctionem*

<TRACTATUS PARISIENSIS DE ENUNTIABILIBUS I>[1]
Paris BNF, lat. 3713, fols 86r–87r

| 86r | **§1** *[Definition and eternity of assertables]* Cum inter universa secundum dialecticam facienda, probare aliquid vel improbare sit potissimum, ad maiorem huius artis cognitionem quae, qualiter et qua ratione probari habeant expedire proponimus. Hoc autem certius innotescet, prius tamen cognito quid ad quid sequi habeat et quid non. Omne enim illud per quod aliud et quod per aliud probatur, <et> quod ad aliud et quod per aliud sequitur, enuntiabile appellamus. Enuntiabile vero enuntiatione significabile vel dicibile. Omne autem enuntiabile ab aeterno fuit, et est, et in aeternum erit; et ex necessitate habet esse, ut quod <nec> habuit principium nec habebit finem.

§2 *[First division of assertables into kinds]* Horum aliud verum, aliud falsum. Et si quid est enuntiabile, est verum vel falsum, et econverso. Eorundem etiam aliud possibile, aliud impossibile. Et si quid est

[1] The entire *Tractatus* is contained in fols 86r–88v; this Appendix only contains the first half of the treatise. For the preparation of this partial edition of the first *Tractatus Parisiensis de enuntiabilibus*, I was able to take advantage of Yukio Iwakuma's private transcription of the same text and the advice of Caterina Tarlazzi and Wojciech Wciórka. All remaining errors are mine. The punctuation and division into paragraphs have sometimes been modified with respect to those present in the manuscript. Paragraph numbers and annotations within square brackets are my own. Italicised expressions indicate assertables.

enuntiabile, est possibile vel impossibile, et econverso. Est autem enuntiabile possibile quod potest esse verum, et si aliquod enuntiabile potest esse verum, est possibile, et econverso. Est autem enuntiabile impossibile quod non potest esse verum, et si quid est enuntiabile quod non potest esse verum, est impossibile, et econverso. Verorum aliud necessarium, aliud non necessarium. Est autem verum necessarium, ut ait Boethius, quod ita est ut dicitur nec aliter esse potest, id est, quod est verum et non potest esse falsum. Verum non necessarium est verum quod potest esse falsum. Necessariorum aliud per se, aliud per accidens. Necessarium per se est verum quod nec potest nec potuit nec poterit esse falsum, ut *Aliquid esse*; necessarium per accidens <est> quod, cum sit necessarium, potuit esse falsum, ut *Me fuisse*. Eodem modo impossibilium aliud per[1] se, aliud per accidens. Est autem impossibile per se quod nec potuit nec potest nec poterit esse verum, ut *Hominem esse asinum*; impossibile per accidens quod, cum sit falsum, potuit esse verum, ut *Me non fuisse*.

§3 *[Definition of good consequence; inferential relations between kinds of assertables]* Dicto quae et quot sint enuntiabilium genera, quid eorum ad quid[2] sequi habeat consequens est dicere. Ex omni igitur enuntiabili id sequi dicitur sine quo[3] ipsum nec potest nec potuit nec poterit esse verum. Si igitur duo sint enuntiabilia quorum primum nec potest nec potuit nec poterit esse verum sine reliquo, ad primum sequitur secundum. Verbi gratia: *Socratem esse aliquid* nec potest nec potuit nec poterit esse verum sine hoc, *Socratem esse hominem*, quare ad *Socratem esse aliquid* sequitur *Socratem esse hominem*. Econtrario, si duo sunt enuntiabilia quorum primum potest vel potuit vel poterit esse verum sine secundo, ad primum non sequitur secundum. Verbi gratia, *Deum esse* potest esse verum sine hoc, *Deum*[4] *modo esse*; non ergo, si Deus est, Deus modo est. Omnis enim consequentia <est> vera cuius antecedens nec potest nec potuit nec poterit esse verum sine consequente. Econtrario, omnis consequentia est falsa cuius antecedens potest vel potuit vel poterit esse verum sine

[1] per] *lineam verticalem supra P scripsit MS*
[2] Quid ad quid] *corr.*, quod ad quod *MS*
[3] *post* quo] nec *delevit MS*
[4] *post* Deum] d *delevit MS*

consequente. Ex omni vero sequitur verum et non nisi verum. Ad quod datur haec regula: si aliquid sequitur ad aliud et primum est verum, et secundum. Ex falso sequitur et verum et falsum. Ex falso possibili nonnisi possibile. Ad quod datur haec regula: si aliquid sequitur ad aliud et primum est possibile, et ultimum. Ex falso impossibili sequitur et possibile et impossibile.

§4 *[Further division of necessary and impossible assertables into* per se *and* per accidens *ones; everything follows from what is* per se *impossible; what is* per se *necessary follows from anything]* Sed ut ista clarius innotescant, attende quoniam verorum et falsorum duo sunt genera: verum necessarium per se, falsum[1] impossibile per se. Ex quorum uno sequitur omne enuntiabile, et quorum unum[2] sequitur ex omni enuntiabili. Ex omni igitur impossibili[3] per se, et eo solo, omne sequitur enuntiabile. Omne necessarium per se, et ipsum solum, ex omni sequitur enuntiabili. Quod qualiter fieri possit ratione demonstrandum. Et prius quod ex omni impossibili per se omne sequatur enuntiabile determinandum.

§5 *[Division of impossibility* per se *into six kinds; the first and main kind]* Sunt igitur impossibilium per se genera plurima, de quorum singulis singillatim disseramus. Unum et principale per[4] se impossibilium genus <est> cum dicitur res unius speciei esse res alterius, ut *Socrates est asinus*. Ex quo omne sequi enuntiabile liquere[5] poterit, prius cognito quod si Socrates est asinus, Socrates est homo. Ad quod ratio incontradicibilis est haec: *Socratem esse aliquid, Socratem esse hominem* duo sunt vera, quorum neutrum potest vel potuit vel poterit esse verum sine altero, et ad unum sequitur alterum, ergo si unum est verum, et alterum. Ergo, si verum est *Socratem esse aliquid*, verum est *Socratem esse hominem*. Ergo, si Socrates est aliquid, Socrates est homo.

[1] *post* falsum] necessarium *delevit MS*
[2] unum] unum unum *MS*
[3] impossibili] impossibile *MS*
[4] per] *lineam supra P add. MS*
[5] liquere] lequere *MS*

Item, si Socrates est aliquid[1], aliquid est Socrates, et si aliquid est Socrates, ipsum est homo[2]. Ergo, si Socrates est aliquid, aliquid est Socrates et ipsum est homo. Ergo, si Socrates est aliquid, Socrates est homo.

Item, si Socrates est aliquid, Socrates est homo vel aliud quam homo. Sed, si aliquid est Socrates, ipsum non est aliud quam homo. Ergo, si Socrates est aliquid, Socrates est homo.

Item, si aliquid est[3] Socrates, illud est homo, et si aliquid est Socrates, idem est Socrates. Ergo, si aliquid est Socrates, aliquid est homo et idem est Socrates. Ergo, si aliquid est Socrates, homo est Socrates.

Item, si aliquid est Socrates, ipsum est hic homo, et econverso; si Socrates est aliquid, hic homo est aliquid, et econverso. Ergo, si Socrates est aliquid, Socrates est hic homo et econverso. Ergo, si Socrates est aliquid, Socrates est homo.

| 86v | Hoc ergo constante, quod si Socrates est aliquid, Socrates est homo, et si est animal est homo, facile[4] est ad hoc transire quod si Socrates est asinus, Socrates est homo. Si enim Socrates est aliquid, Socrates est homo, et si quid est asinus, est aliquid. Ergo, si Socrates est asinus, Socrates est homo.

§6 *[Against the opinion of some according to whom nothing follows from false assertables, and false assertables follow from nothing]* Sunt tamen nonnulli perversi disputatores qui[5] verentur huiusmodi concedere: si Socrates est asinus, Socrates est aliquid. Negant enim ex falso sequi aliquid, vel[6] falsum sequi ex aliquo. Contra quos hic ex incidenti haec obicienda insinuamus: si Socrates est animal, Socrates est rationale vel irrationale. Ergo, si Socrates est animal et non est rationale, est irrationale.

Item, ad *Socratem esse asinum* nihil sequitur: ad *Socratem esse hominem* sequitur *esse rationale*. Ad *Socratem esse hominem vel asinum*

[1] *post* aliquid] Socrates *delevit MS*
[2] homo] Socrates *a.c.*
[3] *post* est] hoc *vel* homo *delevit MS*
[4] facile] facilis *MS*
[5] qui] quod *MS*
[6] *post* vel] aliquid ex *delevit MS*

aliquid sequitur. Ergo, ad *Socratem esse hominem vel asinum* sequitur *esse rationale*.

Item, si possibile est *Socratem esse sanum*, possibile est *Socratem esse animal*. Ergo, vel si verum est *Socratem esse sanum*, verum est *Socratem esse animal*, vel si falsum est *Socratem esse sanum*, falsum est *Socratem esse animal*, vel non si aliquid est sanum, ipsum est animal.

§7 *[Proof that every assertable follows from the per se impossible "Socrates is a donkey"]* Viso qualiter ad *Socratem esse asinum*[1] sequatur *Socratem esse hominem*[2], quomodo ad idem omne sequatur enuntiabile est videndum: si Socrates est asinus, Socrates est homo. Ergo, si Socrates est asinus, Socrates est homo vel omne enuntiabile est verum. Sed si Socrates est asinus, Socrates non est homo. Ergo, si Socrates est asinus, omne enuntiabile est verum. Ad huiusmodi argumentum confirmandum, datur haec regula: quotiens duo sequuntur ad idem sub disiunctione, si contradictio unius sequitur ad antecedens, reliquum sequitur ex eodem.

§8 *[The second kind of impossibility per se]* Est aliud[3] impossibilium genus per se, quotiens dicitur de re[4] unius speciei quod sit res duarum utriusque, ut cum dicitur *Socratem esse et hominem et equum*. Ex quo omne sequi enuntiabile sic ostendimus: si Socrates est et homo et equus, Socrates est homo, et si est homo, non est equus (locus ab oppositis). Et si non est equus, Socrates non est et homo et equus. Ergo, a primo, si Socrates est et homo et equus, Socrates non est et homo et equus. Ergo, si Socrates est et homo et[5] equus, Socrates <non> est et homo et equus vel omne enuntiabile[6] est verum. Sed, si Socrates est et homo et equus, falsum est quod Socrates non est et homo et equus. Ergo, si Socrates est et homo et equus, omne enuntiabile est verum. Similiter sequeretur, si Socrates est et homo et equus, Socrates est et vacca et archiepiscopus.

[1] asinum] hominem *a.c. MS*
[2] hominem] asinum *a.c. MS*
[3] Aliud *iteravit MS*
[4] re] rei *a.c. MS*
[5] et] *supra lineam MS*
[6] enuntiabile] equntiabile *MS*

§9 *[The third kind of impossibility per se]* Est tertium per se impossibilium genus, quotiens de uno singulari dicitur ipsum esse aliud singulare, ut cum dicitur *Socratem esse Platonem*. Quod, si verum est, Socrates est capra cornuta, quia: si aliquid est Socrates, ipsum non est Plato, et si aliquid est Plato, ipsum non est Socrates (locus ab oppositis). Ergo, si Socrates est Plato, Socrates non est Plato. Ergo, si Socrates est Plato, Socrates non est Plato vel Socrates est capra cornuta. Sed, <si> Socrates est Plato, falsum est quod Socrates non sit Plato (locus a contradictorie oppositis). Ergo, si Socrates est Plato, Socrates est capra cornuta. Similiter, si Socrates est capra, omne enuntiabile est verum.

§10 *[The fourth kind of impossibility per se]* Est quartum per se impossibilium genus[1], cum dicitur contradictorie oppositorum utrumque esse verum; ut cum dicitur: *Socratem esse*, *Socratem non esse*. Quod, si verum est, omne enuntiabile est verum. Si enim haec sunt contradictorie opposita, et unum est verum, reliquum est falsum, et econverso: talis enim est natura et lex contradictorie oppositorum. Ergo, si haec sunt contradictorie opposita, non utrumque est verum. Ergo, si haec sunt contradictorie opposita et utrumque est verum, non utrumque est verum. Ergo, si haec sunt contradictorie opposita et utrumque est verum, non utrumque <est> verum vel omne enuntiabile est verum. Sed si haec sunt contradictorie opposita et utrumque est verum, falsum est quod horum non utrumque est verum. Ergo, si haec sunt contradictorie opposita et utrumque est verum, omne enuntiabile est verum.

Circa huiusmodi concedunt quidam quod contradictorie oppositorum utrumque esse verum est possibile. Contra quos sic argumentandum: utrumque horum—*te legendum*, *te non legendum*—esse verum est possibile. Et si aliquorum utrumque est verum, neutrum eorum est falsum. Ergo, possibile est neutrum horum esse falsum. Ergo, non est necesse alterum horum esse verum et alterum esse falsum. Ergo non sunt contradictorie opposita. Contradictorie igitur oppositorum utrumque interdum est possibile, et utrumque possibile est esse verum. Numquam

[1] *post* genus] d *delevit MS*

possibile est contradictorie oppositorum utrumque esse verum nec umquam contradictorie oppositorum utrumque esse verum est possibile.

§ 11 *[The fifth kind of impossibility per se]* Est et quintum per se impossibilium genus, quotiens contrarium de contrario innuitur praedicari, licet utrumque possit praedicari de quo reliquum, ut cum dicitur *Album esse nigrum*. Quod si verum est, Socrates est asinus, quia si aliquid est album, ipsum non est nigrum, et si aliquid est nigrum, ipsum non est album. Ergo, si aliquid album est nigrum, ipsum non est nigrum. Ergo, si aliquid album est nigrum, ipsum non est nigrum vel Socrates est asinus. Sed, si album est nigrum, falsum est quod ipsum non est nigrum. Ergo, si album est nigrum, Socrates est asinus. Similiter, si album est nigrum, omne enuntiabile est verum.

Contra hos qui dicunt quod possibile sit album esse nigrum, haec est ratio. Si aliquid est verum, ipsum est verum. Ergo, si aliquid est verum, nihil est <verum>[1] quod non sit verum. Ergo, si aliquid est verum, nihil est verum quod sit falsum[2]. Sed necessarium est aliquid esse verum. Ergo, necessarium est nihil esse verum[3] quod sit falsum. Ergo, impossibile est aliquid quod est verum esse falsum.

Item, nihil simul est album et nigrum. Ergo, nihil aliquando est album quando ipsum sit nigrum. Hoc necessarium *Nihil simul esse album et nigrum*, ergo necessarium est[4] nihil aliquando esse album quando ipsum sit nigrum. Ergo, impossibile est aliquid esse album quando ipsum est nigrum. | 87r | Praeterea quidlibet necessarium est vel non esse album vel non esse nigrum. Ergo, quidlibet impossibile est et esse album et esse nigrum. Ergo, quidlibet impossibile est esse et album et nigrum.

§12 *[The sixth kind of impossibility per se]* Est et sextum per se impossibilium genus, quotiens innuitur aliquid quod inest alicui non inesse eidem, ut cum dicitur: *Id quod est album non esse album*; *Id quod sedet non sedere*. Ad quod constat sequi omne enuntiabile, et similia. Quod sic probamus: si aliquid est Socrates, et illud est vel non est, ipsum est. Ergo,

[1] verum] *addidi Iwakuma probante*
[2] *post* falsum] ergo si *delevit MS*
[3] verum] *supra lineam add. MS*
[4] est] *supra lineam add. MS*

si id, quod est Socrates, est vel non est, ipsum est. Ergo, si id, quod est Socrates, non est, ipsum est. <Ergo, si id, quod est Socrates, non est, ipsum est vel omne enuntiabile est verum. Sed si id, quod est Socrates, non est, falsum est quod ipsum est. Ergo, si id, quod est Socrates, non est, omne enuntiabile est verum.>

Similiter, si aliquid legit et ipsum legit vel non legit, ipsum legit. Ergo, si id, quod[1] legit, legit vel non legit, ipsum legit. Ergo, si id, quod legit, non legit, ipsum legit. Ergo, si id, quod legit, non legit, ipsum legit vel omne enuntiabile est verum. Sed, si id, quod legit non legit, falsum est quod ipsum legit. Ergo, si id, quod legit non legit, omne enuntiabile est verum.

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