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Sign and Demonstration in Late-Ancient Commentaries on the *Posterior Analytics*

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Introduction

In chapter 6 of book I of the Posterior Analytics, Aristotle contrasts proper demonstrations, which proceed from the cause, with syllogisms through signs (οί διὰ σημείων συλλογισμοί), which do not proceed from the cause (75a31-34). An analogous contrast is drawn in chapter 17 of book II (99a1-4). Considering that in the Prior Analytics, in the context of the 'official' exposition of his doctrine of sign-arguments in chapter 27 of the second book, Aristotle defines a sign ($\sigma\eta\mu\epsilon\tilde{i}\sigma\nu$) as a 'demonstrative premise' ($\pi\rho \delta \tau \alpha \sigma \sigma c \alpha \pi \delta \delta c \kappa \tau \kappa \dot{n}$) (70a6–7), the claim that a syllogism through signs is not a demonstration might appear somewhat surprising. Maybe this is the reason why it was targeted by ancient commentators.¹ Aristotle's scanty remarks at APo. I.6 and II.17 about sign-arguments form the backbone of a doctrine which, as we shall show, was first set forth by Alexander of Aphrodisias and then re-stated by Themistius and Philoponus—a doctrine according to which the 'syllogism of the that' discussed by Aristotle at APo. I.13 is in fact a syllogism from a sign, and consequently APo. I.6 and II.17 ought to be read as contrasting proper demonstration, or demonstration 'of the why', with a second standard of demonstration, associated with the syllogism of the that or 'from a sign'. The identification was made both implicitly, through the employment of Aristotle's examples of sign-syllogisms from the Prior Analytics and the *Rhetoric* as examples of syllogisms of the that, and explicitly, thus becoming fully integrated into the commentary tradition.²

¹ The contrast between sign and demonstration drawn in the *Posterior Analytics* has received little attention in the recent scholarly literature. In his detailed commentary, Mignucci devotes one short paragraph to this issue (1975: 134), while Barnes dismisses it with a cursory reference to *APr*. II.27 (1993: 130). But see Allen (2001: 72–78) and Manetti (1993: 87). See also Bellucci (2018) for an expanded analysis of the content of §1 of the present paper.

² The identification of the syllogism from signs with the syllogism of the that is found in Robert Grosseteste (*In APo.*, II, 2), Robert Kilwardby (*In APo.*, P, f. 135v, <lemma 20>),

This paper offers an exposition and a critical examination of the ancient interpretations of Aristotle's contrast between sign and demonstration in the context of the theory of scientific demonstration expounded in his *Posterior Analytics*. It is divided into four sections, devoted to Aristotle, Alexander, Themistius, and Philoponus, respectively.

1. Aristotle

Scientific knowledge (ἐπιστήμη) is knowledge through demonstration (ἀπόδειξις), and a demonstration is a kind of syllogism (συλλογισμός τις, *APr*. I.4, 25b30), namely the scientific syllogism (συλλογισμός ἐπιστημονικός, *APo*. I.2, 71b18), the syllogism used to produce (or to impart¹) ἐπιστήμη.

At APo. I.13 Aristotle distinguishes between scientifically knowing 'the that' (τὸ ὅτι ἐπίστασθαι) and scientifically knowing 'the why' (τὸ διότι ἐπίστασθαι). Since scientific knowledge is knowledge by demonstration, and a demonstration is a scientific syllogism, the two kinds of scientific knowledge correspond to two kinds of scientific syllogisms, the 'syllogism of the that' (συλλογισμός τοῦ ὅτι) and the 'syllogism of the why' (συλλογισμός τοῦ διότι). The syllogism of the that differs from the syllogism of the why both in different sciences and within the same science. Within the same science, the distinction is in turn twofold. First (78a23–26), when the syllogism proceeds from premises which are not immediate: in this case, we have a syllogism of the that but not of the why. Second (78a26–29), when the syllogism does proceed from premises which are immediate, but infers the cause ($\tau \dot{o} \alpha \dot{\tau} \tau \sigma v$) from the effect ($\tau \dot{o}$ μὴ αἴτιον, 78a29, τὸ ἀναίτιον, 78b11). It is this second difference between scientifically knowing the that and scientifically knowing the why within the same science that will concern us.²

Aristotle considers a situation in which two terms are related to one another as cause and effect, where the effect is better known

Albert the Great (*In APo.*, Tract. II, ch. XVI), and Thomas Aquinas (*In APo.*, 14, 6). We also find traces of this exceptical thread in the ps-Kilwardby (*Super Priscianum maiorem*, I, 1, 1 [3]) and Roger Bacon (*De signis*, 1, 6).

¹ According to Barnes (1969), the theory of demonstrative science presented in the *Posterior Analytics* was not meant to describe how scientists do, or ought to, acquire knowledge. It was meant to describe how teachers should impart knowledge. For a discussion of his view see Burnyeat (1981: 115–120).

² For the case discussed at 78b23–26, see Ross (1949: 495–496), Barnes (1993: 155–156), and Mignucci (1975: 294–297).

(γνωριμώτερον) than the cause. There are supposedly two possible cases: case I (examined at 78a23–78b11), in which cause and effect are convertible terms (ἀντικατηγορουμένων), and case II (examined at 78b11–13), in which they are not convertible.

Case I. Convertible cause and effect. Suppose that, in an astronomical context, the fact that something is near to the earth is the cause of its not twinkling. Not twinkling and being near are convertible terms: that which is near does not twinkle and that which does not twinkle is near. At the same time, something's not twinkling is better known than its being near to the earth, because the former is directly accessible to perception while the latter is not. Given these assumptions, consider the following syllogism:

(1)

That which does not twinkle is near Planets do not twinkle Hence, planets are near

Here the cause ('being near') is inferred from the effect ('not twinkling'). The syllogism that infers the cause, or the less known, from the effect, or the better known, is a syllogism of the that. But since cause and effect are convertible terms, another syllogism is possible:

(2)

That which is near does not twinkle Planets are near Hence, planets do not twinkle

Here, the effect ('not twinkling') is inferred from the cause ('being near'). The syllogism that infers the effect, or the better known, from the cause, or the less known, is a syllogism of the why. Both are valid syllogisms in the first figure; only, (1) infers the cause from the effect, (2) the effect from the cause. It has to be noted that, according to the distinction of *APo*. I.2, a cause is less known to us but is better known in itself, and an effect is better known to us but less known in itself. Thus in a syllogism of the why something better known to us is inferred from something better known in itself is inferred from something better known in itself is inferred from something better known to us.

Case II. Non-convertible cause and effect. At 78b11-13 the case in which cause and effect do not convert is briefly presented. Aristotle only savs: 'where the middle terms do not convert and the non-cause ($\tau \dot{o}$) $\dot{\alpha}$ v α (trans.) is more familiar, the fact is proved but not the reason why' (trans.) Barnes, modified). There are potentially two sub-cases of case II: sub-case II.a. in which the cause has wider extension than the effect (the occurrence of the effect implies the occurrence of the cause, but the occurrence of the cause does not imply the occurrence of the effect), and sub-case II.b, in which the effect has wider extension than the cause (the occurrence of the cause implies the occurrence of the effect, but the occurrence of the effect does not imply the occurrence of the cause). Sub-case II.a is one in which an effect may or may not be produced by the cause, and the cause is validly inferable from the effect, but not the other way round; if a demonstration has to be a valid syllogism, sub-case II.a only admits of a demonstration of the that. Sub-case II.b is one in which a cause produces an effect which could also be produced by another cause, and thus the effect is validly inferable from the cause, but not the other way round; if a demonstration has to be a valid syllogism, sub-case II.b only admits of a demonstration of the why.

Now, it is clear that the sub-case of non-converting effect and cause that Aristotle has in mind at 78b11–13 is sub-case II.a, i.e., that in which the cause has wider extension than the effect. For if sub-case II.b were under examination here, i.e., the case in which the effect has wider extension than the cause, then Aristotle should say that in such a case only a syllogism of the why would be possible (for the effect is validly inferable from the cause), but not one of the that (for the cause is not validly inferable from the effect). The fact that Aristotle says that the syllogism in this case can only be of the that but not of the why doubtless shows that he has sub-case II.a in mind.¹

No example is offered by Aristotle of a sub-case-II.a syllogism, but Barnes (1993: 156–157) has supplied one:

The wallaby has a pouch All pouched animals are mammals

¹ There seems to be little ground for McKirahan's claim that in the case discussed at 78b11–13 '[o]nly the proof (corresponding to [the syllogism of the why discussed at 78a40–78b3]) can be formed' (1992: 224). For Aristotle says just the opposite, i.e., that in this case of non-converting cause and effect, only a syllogism of the that is possible. McKirahan appears to conflate sub-case II.a with sub-case II.b.

Therefore the wallaby is a mammal

Being a mammal is the cause of having a pouch, but having a pouch and being a mammal do not convert, because while all marsupials are mammals, not all mammals are marsupials; the effect, that wallabies have pouches, is better known than the cause, their being mammals. The syllogism is consequently of the that, because the cause (being a mammal) is validly inferred from the effect (having a pouch). But there *cannot be* a valid syllogism of the why, because the cause has wider extension than the effect (not all mammals have a pouch).

According to *APo.* I.13, a syllogism of the that is a valid syllogism which either infers the cause from an effect convertible with it (I.a), or infers the cause from an effect which is less extended than its cause (II.a). Now, ancient and medieval commentators on the *Posterior Analytics* have variously called the syllogism of the that in *APo.* I.13—of either type I.a or II.a—a 'syllogism through signs'. In fact, Aristotle himself seems to suggest such an identification at *APo.* I.6, where he contrasts demonstration proper with syllogisms through signs. Yet, *APo.* I.13, where the distinction between the syllogism of the that and the syllogism of the why is first presented, does not mention signs.

What is a 'sign' in Aristotle? The official presentation of the doctrine of signs is at *APr*. II.27. This chapter is about enthymemes, and an enthymeme is a syllogism starting from (i.e., whose premises are either) probabilities or signs ($\dot{\epsilon}\xi \epsilon i\kappa \dot{\sigma} \omega v \ddot{\eta} \sigma \eta \mu \epsilon i\omega v$, II.27, 70a10).¹ A sign is thus the premise of an enthymeme:

A sign, however, is supposed to be either a necessary or an accepted demonstrative premise ($\pi\rho \circ \tau \alpha \sigma \iota \varsigma \ \alpha \pi \circ \delta \varepsilon \iota \kappa \tau \iota \kappa \eta \ \eta \ \alpha \nu \alpha \gamma \kappa \alpha \tilde{\iota} \alpha \ \eta \ \tilde{\varepsilon} \nu \delta \circ \xi \circ \varsigma$). For whatever is such that if it is, a certain thing is, or if it happened earlier or later, the thing in question would have happened, that is a sign of this thing's happening or being (II.27, 70a6-9; trans. Smith).

¹ In the *Rhetoric* enthymemes are classified according to the thing they are based on: probability (εἰκός), example (παράδειγμα), necessary sign (τεκμήριον), and fallible sign (σημεῖον) (*Rhet.* II.25, 1402b). At *Rhet.* I.2, 1356b4–5 the example is made coordinate with the enthymeme rather than one of its sources and species. Examples are rhetorical inductions; enthymemes are rhetorical syllogisms. Following Ross (1949: 500), if we exclude examples, and if we note that both τεκμήρια and σημεῖα in the strict sense are σημεῖα in the wide sense, we see that the classification of *Rhet.* I.2 corresponds to that of *APr.* II.27.

A sign is a demonstrative premise ($\pi\rho \circ \tau \alpha \sigma \iota \varsigma \ \dot{\alpha} \pi \circ \delta \varepsilon \iota \kappa \tau \iota \kappa \dot{\eta}$). Here a first difficulty arises: if being an $\dot{\alpha} \pi \circ \delta \varepsilon \iota \kappa \tau \iota \kappa \dot{\eta}$ premise means being the premise of an $\dot{\alpha} \pi \circ \delta \varepsilon \iota \varsigma \iota \varsigma ,$ the claim at *APr*. II.27 that a sign is an $\dot{\alpha} \pi \circ \delta \varepsilon \iota \kappa \tau \iota \kappa \dot{\eta}$ premise contrasts with what *APo*. I.6, as we shall see shortly, says of syllogisms through signs, i.e., that they are *not* demonstrations. After all, a sign is the premise of one kind of enthymeme, and an enthymeme is a *rhetorical* syllogism (*Rhet*. I.2, 1356b4), i.e., the syllogism used in rhetorical discourse to produce persuasion, not a *scientific* syllogism, i.e., the syllogism used to produce or impart scientific knowledge. As we shall see, commentators on the *Posterior Analytics* clearly perceived the difficulty, which they solved by distinguishing a demonstration in the proper sense from a demonstration in a secondary sense: if a sign-inference is to be a demonstration, it can be so only in a secondary sense.

Since signs are premises of enthymemes, and enthymemes are syllogisms, signs are analyzable by means of the formal apparatus of syllogistic. As there are three syllogistic figures, depending on the position of the middle term in the premises, so there must be three kinds of semiotic enthymemes (*APr*. II.27, 70a11–23). Here are Aristotle's examples:¹

First figure This woman (C) has milk (B) Whoever has milk (B) has born a child (A) This woman (C) has born a child (A).

Second figure This woman (C) is pale (A) Whoever has born a child (B) is pale (A) This woman (C) has born a child (B)

Third figure *Pittakos* (C) *is good* (A)

¹ We translate the verb κύειν (usually rendered by 'being pregnant') with 'having born a child' on the base of its reception and interpretation by Greek commentators, such as Alexander of Aphrodisia and Philoponus (see below, §2 and §4), and Latin translators, such as Boethius (who used the verb *parĕre*, see AL 3.1: 137). This translation better fits the first example and matches with *Rhet*. 1357b15–16, where the verb used is τέτοκεν, having born a child. It has yet to be noted that Aristotle typically calls γάλα both the milk and the colostrum (cf. Burnyeat 1982: 195n30) and that in the *Gen. an*. IV, 776a20–b4 he explains that milk is produced in the woman's breast by the seventh month of pregnancy.

Pittakos (C) *is wise* (B) *Wise people* (B) *are good* (A)

The sign-syllogism in the first figure is 'irrefutable' ($\ddot{\alpha}\lambda \upsilon \tau o \zeta$, 70a29–30), i.e., necessary or deductively valid, while sign-syllogisms in the second and third figure are 'refutable' ($\lambda \dot{\omega} \sigma \iota \mu o \iota$, 70a31, 34), i.e., deductively invalid. Aristotle calls $\tau \epsilon \kappa \mu \dot{\eta} \rho \iota o \upsilon$ the sign in the first figure and $\sigma \eta \iota \epsilon \ddot{\alpha}$ in the strict sense the signs in the second and third figure. T $\epsilon \kappa \mu \dot{\eta} \rho \iota \alpha$ are evidences, necessary signs; $\sigma \eta \iota \epsilon \ddot{\alpha}$ in the strict sense are indications, non-necessary signs. $\Sigma \eta \iota \epsilon \ddot{\alpha}$ in the wide sense (registered at 70b1) include both $\tau \epsilon \kappa \mu \dot{\eta} \rho \iota \alpha$ and $\sigma \eta \iota \epsilon \ddot{\alpha}$ in the strict sense (registered at 70b1). At *Rhet.* I.2, 1357b, $\sigma \eta \iota \epsilon \ddot{\alpha}$ in the strict sense are said to require no specific name. We may say that while $\sigma \eta \iota \epsilon \ddot{\omega}$ is the unmarked term of the opposition, $\tau \epsilon \kappa \mu \dot{\eta} \rho \iota o \nu$ is the marked one.

The syllogistic reconstruction of sign-inferences allows Aristotle to sharply differentiate first figure signs that are irrefutable or deductively valid (ἄλυτα σημεῖα or τεκμήρια) from second and third figure signs that are refutable or deductively invalid (λύσιμα σημεῖα or σημεῖα in the strict sense).¹ Having milk is a τεκμήριον of having born a child, but being sallow is only a σημεῖον (in the strict sense) of the same thing.

Now, an effect is a sign of its cause, and thus we may say that a syllogism of the that infers a cause from a sign of it. In light of the distinction of *APo*. I.13 between *different varieties of syllogisms of the that*, we can say—the commentators have in fact said as much, as we shall see—that a $\tau \epsilon \kappa \mu \eta \rho \omega$ is a syllogism of the that of sort I.a (cause and effect convert, and the cause is validly inferred from the effect) or II.a (cause and effect do not convert, the cause has wider extension than, and thus is validly inferable from, the effect), while a $\sigma \eta \mu \epsilon \omega$ in the strict sense is the premise of a syllogism of sort II.b (cause and effect do not convert, the effect has wider extension than, and thus from it we cannot validly infer,

¹ As noticed by Morrison (1997: 4–5), the inferences from signs of *APr*. II.27 feature singular terms (this woman, Pittakos), and this fits awkwardly in the context of *APr*., for Aristotle excludes singular terms and singular propositions from his systematic discussions of syllogistic form (cf. *APr*. I.27, 43a23–45). But as Burnyeat has persuasively argued, *APr*. II.27 'should not be listed as an exception to the exclusion of singular terms from syllogistic' (1982: 195n7). *APr*. II.27 is not supposed to adhere strictly to the formal syllogistic theory previously expounded. That chapter is only presented as an explication of how common reasoning from signs, in which singular terms and singular propositions frequently occur, can be reconstructed in terms of syllogistic theory, even at the price of some stretching of the theory.

the cause). A syllogism of the that of sort I.a and II.a is a deductively valid argument that infers the cause from the effect ($\tau \epsilon \kappa \mu \eta \rho \iota \sigma \nu$), whereas a syllogism of sort II.b is a deductively *invalid* argument that infers the cause from the effect ($\sigma \eta \mu \epsilon \tilde{\iota} \sigma \nu$) in the strict sense).

Since syllogistic is a test of logical validity, Aristotle is able by means of it to differentiate valid from invalid sign-inferences. But it would be wrong to maintain that Aristotle's aim at *APr*. II.27 is to straightforwardly reject sign-inferences that do not admit of a deductively valid reconstruction in syllogistic terms. His aim is rather to recognize how sign-inferences are related to the syllogism and how the distinction between the syllogistic figures allows a corresponding distinction between the grades of evidential support that signs provide. Aristotle expressly maintains that each sign-inference is conducive to truth in its own way: 'the truth, then, can occur in all signs, but they have the differences stated' (*APr*. II.27, 70a37–38).¹ Whether deductively valid or not, a sign-inference is leading to truth in its own way. However, neither the valid nor the invalid variety can qualify as a demonstration in the proper sense, as we shall now proceed to explain with the help of the two passages of the *Posterior Analytics* that mention inferences from signs.

In contrast to the definition of APr. II.27, according to which a sign is a demonstrative premise (i.e., the premise of a demonstration), at APo. I.6 we are told that a syllogism through signs is *not* a demonstration: 'what is incidental is not necessary, so that you do not necessarily know why the conclusion holds (οὐκ ἀνάγκη τὸ συμπέρασμα εἰδέναι διότι ὑπάργει)—not even if it is the case always but not in itself, as for example in syllogisms through signs (oùd' ei del ein, $\mu\eta$ καθ'αυτό δέ, οἶον οι διὰ σημείων συλλογισμοί)' (75a31-34). According to APo. I.2, the premises of a demonstration must be true, primitive, and immediate, and must be better known than, prior to, and cause of, the conclusion (71b21-22). Now, the fact that the premises of a demonstration must be or contain the cause of the conclusion entails that an accidental predication cannot constitute the basis of a demonstration: assuming that the conclusion of a demonstration is necessary, the premises must be necessary too. For were they not necessary, they would be accidental, and something accidental cannot be the cause of something necessary (74b27-32). Recall Aristotle's claim at 75a31-32: since an accidental proposition cannot contain the cause of a necessary proposition, then one who knows the truth of such a conclusion

¹ Cf. Burnyeat (1982: 195–197).

would not thereby know *why* it is true, for knowing the reason of the truth of a proposition is to know its cause.

The contrast drawn at 75a31–34 between demonstration and syllogism through signs therefore assumes that a demonstration through signs is not a causal demonstration. That is, a syllogism through signs does not infer the effect from the cause, but only the cause from the effect. In the terminology of *APo*. I.13, a syllogism through signs is a syllogism of the that. But in light of the distinction of *APo*. I.13, a syllogism of the that can be of two kinds: I.a and II.a, both of which are deductively valid (τεκμήριον).

Now, if the term σημεῖον at APo. I.6, 75a33 is taken in the strict sense registered at APr. II.27, 70b4, and not in the wide sense of 70b1, then in that context the contrast would be between a syllogism of the why and an *invalid* syllogism of the that (II.b). But Aristotle's phrase oùo'ei dei είη, μὴ καθ'αὐτὸ δέ ('not even if it is the case always but not in itself') seems to suggest that what is at stake here is rather a syllogism which, though deductively valid (the conclusion 'is the case always'), does not qualify as a demonstration, because it does not satisfy one of the requirements of I.2, namely that the premises must be cause of the conclusion (what is predicated καθ'αύτὸ is necessary; but nothing accidental can be the cause of something necessary, as the conclusion of the syllogism is required to be according to 75a31-34). If this interpretation is correct, then, the term $\sigma \eta \mu \epsilon \tilde{i} \sigma v at 75a33-34$ is used in the wide sense of APr. II.27, 70b1 (corresponding to a syllogism of the that of sort I.a or II.a), not in the strict sense of 70b4 (corresponding to II.b). If the distinction between $\tau \epsilon \kappa \mu \eta \rho \alpha$ and $\sigma \eta \mu \epsilon \tilde{\alpha}$ had been at his disposal,¹ then in order to be more precise, in that context Aristotle should have talked of τεκμήρια, not of σημεῖα (even though in the wide sense), and should have rather contrasted demonstrations with συλλογισμοί διά τεκμηρίων. As we shall see below ($\S4$), Philoponus seems to fully realize this, and in his commentary on the first book of the Posterior Analytics prefers to contrast demonstration proper with what he calls τεκμηριώδης άπόδειξις, 'tekmeriodic demonstration' (Philoponus, In APo., 49, 12; 169, 8; 386, 31). A tekmeriodic demonstration is a syllogism of the that which is deductively valid, otherwise it would not be a demonstration at all.

¹ Here is a further argument in favor of the thesis, defended by Solmsen (1929) and Barnes (1981), that parts at least of the *Posterior Analytics* were written before the *Prior Analytics*, and thus ignore the formal theory of syllogistic figures and moods and *a fortiori* the semiotic doctrine based on that theory.

The second and only other mention of sign-inferences in the *Posterior Analytics* occurs at II.17:

Can it or can it not be the case that the cause (α íτιον) of some feature is not the same for every item but different for different items? If the conclusions have been demonstrated in themselves ($\kappa\alpha\theta$ ' α $\dot{\alpha}$ $\dot{\tau}\dot{\alpha}$), and not in virtue of a sign or incidentally (μ $\dot{\eta}$ $\kappa\alpha\tau\dot{\alpha}$ σ η μ εĩον $\ddot{\eta}$ σ υμβεβηκός), then perhaps it is not possible, for the middle term is the account (λ όγος) of the extreme (*APo*. II.17, 99a1–4; trans. Barnes, modified).

A demonstration proper infers the effect from the cause. Now the cause of something is also its definition. For the definition ($\delta\rho\iota\sigma\mu\delta\varsigma$, or, as here, $\lambda\delta\gamma\varsigma\varsigma$) says or shows the what-it-is ($\tau\delta$ τ í $\dot{\epsilon}\sigma\tau\iota$) of something, i.e., what something is (*APo.* II.3, 90b3-4, 30; 91a1; II.10, 93b29; 94a11); the cause is the reason why something is, and to know the cause is to know why something is (*APo.* I.6, 75a35; I.13, 78a27; 78b11–13). What something is and why it is are one and the same thing (*APo.* II.2, 90a15–18). In a proper demonstration, therefore, the middle term is the cause and the definition of the major extreme: the uniqueness of the definition entails the uniqueness of the cause. But if the demonstration is 'through a sign or an accident', the cause is inferred from the effect, and therefore in this case no appeal to the uniqueness of the definition is of any service. Thus, if the demonstration is 'through a sign', nothing guarantees that the cause of any feature will be unique.

This point can be connected with a passage from the *Sophistical Refutations* in which the notion of demonstration from sign ($\kappa \alpha \tau \dot{\alpha} \tau \dot{\sigma}$ $\sigma \eta \mu \epsilon \tilde{i} \sigma v \dot{\alpha} \pi \delta \delta \epsilon \iota \xi \iota \varsigma$) is connected with the fallacy of the consequent:

The refutation which depends upon the consequent arises because people suppose that the relation of consequence is convertible. For whenever, if this is the case, that necessarily is the case, they then suppose also that if the latter is the case, the former necessarily is the case. This is also the source of the deceptions that attend opinions based on sense-perception. For people often supposed bile to be honey because honey is attended by a yellow colour; and since after rain the ground is wet, we suppose that if the ground is wet, it has been raining; whereas that does not necessarily follow. In rhetoric demonstrations from signs ($\kappa \alpha \tau \alpha \tau \delta \sigma \eta \mu \epsilon \delta \alpha \delta \epsilon \xi \epsilon \iota \varsigma$) are based on consequences. For when orators wish to show that a man is an adulterer, they take hold of some consequence—that the man is smartly dressed, or that he is observed to wander about at night. There are, however, many who have these characters, but not the predicate (167b1– 12; trans. Barnes, modified).

In those cases in which the relation between cause and effect is not convertible and the effect has a wider extension than the cause (II.b), the

argument from the effect is a fallacy. Since rain is the cause of the ground's being wet, but not all cases of the ground's being wet are cases of rain, inferring that it has rained from the fact that the ground is wet is a fallacy. In rhetorical syllogisms, i.e., in enthymemes, such an inference has the aspect of a syllogism from a $\sigma\eta\mu\epsilon$ iov in the strict sense. Aristotle's example is in fact a second figure sign-inference, which is deductively invalid:

Adulterers are smartly dressed (or wander about at night) This man is smartly dressed (or wanders about at night) Therefore, this man is an adulterer

The effect (being smartly dressed or wandering about at night) is a sign of the cause (being an adulterer); but inferring the cause from the effect is a fallacy, for there may be other causes of that effect. If the connection with the fallacy of the consequent in *Sophistical Refutations* is correct, then, we may draw the following terminological conclusion: unlike at I.6, 75a33–34, at II.17, 99a3 the term $\sigma\eta\mu\epsilon$ iov is used in the strict sense registered at *APr*. II.27, 70b4 and corresponding to our sub-case II.b.

To sum up, for Aristotle the effect may be a $\sigma\eta\mu\epsilon\tilde{i}ov$ (in the wide sense of 70b1) of its cause in two ways: as a $\tau\epsilon\kappa\mu\eta\rho\iotaov$, when either effect and cause convert (I.a) or the cause is wider than the effect (II.a, *APo*. I.6); or as a $\sigma\eta\mu\epsilon\tilde{i}ov$ (in the strict sense of 70b4), when the effect is wider than the cause (II.b, *APo*. I.17; *SE* V, 167b1–12). With this typology in mind we can now examine Aristotle's commentators.

2. Alexander

There can be little doubt that Alexander of Aphrodisias (c. 200 CE) produced a commentary on the *Posterior Analytics*. In his surviving commentaries, Alexander himself refers several times to the *Posterior Analytics*, either quoting a passage or referring to a doctrine expounded in that work. Also, several manuscripts of the *Posterior Analytics* contain scholia explicitly attributed to Alexander (Moraux 1979: 7), and it is likely that one or more copies of Alexander's commentary, or at least some collection of excerpts based on it, were still available in Constantinople at the beginning of the 12th century, because Eustratius of Nicaea seems to have used it while compiling his own commentary to the *Posterior Analytics* (Moraux 1979: 6). Furthermore, Moraux (1979: 131–135) has persuasively shown that the anonymous commentary on the second book

of the *Posterior Analytics* published by Wallies in CAG 13.3 (547–603), at least in its greater part, is made out of bits and pieces, only slightly modified, of Alexander's lost commentary.¹ In both the anonymous commentary containing excerpts from Alexander and in Alexander's commentary on the *Prior Analytics* the contrast between sign and demonstration emerges with peculiar clarity.

Only the second book of the anonymous commentary survives, though. In commenting on *APo*. II.17 99a1–4 (which contains the second and last mention of sign-inferences in the *Posterior Analytics*) the anonymous says:

λύων δὴ τοῦτο καὶ τὸ διττὸν τοῦ αἰτίου ἐνδεικνύμενος (οὐ γὰρ πάντως τὸ τοῦ συμπεράσματος αἰτιον καὶ τοῦ πράγματός ἐστιν αἰτιον) τοῦτο δὴ δεικνὺς λέγει ὅτι, εἰ μὲν καθ' αὐτὸ ἀποδέδεικται, τουτέστιν εἰ διὰ τοῦ αἰτίου, ὃ τοῦ πράγματός ἐστιν αἴτιον, ἡ δεῖξις εἴη γινομένη, τὸ αὐτὸ αἴτιον ἀνάγκη ἐπὶ πάντων λαμβάνεσθαι οἶς δείκνυται τὸ αὐτὸ ἀπάρχον. οὖ τὴν αἰτίαν παρέθετο, ῆν ἕδειξεν ἤδη, εἰπὼν 'ὁρισμὸς γὰρ τοῦ ἄκρου ἐστὶ τοῦ κατηγορουμένου καὶ τοῖς πλείοσιν ὑπάρχοντος ἡ αἰτία δι' ἦς δείκνυται, καὶ οὖτος ὁ μέσος ὅρος'. ἡ γὰρ τοιαύτη αἰτία κατὰ κοινόν τι καὶ ταὐτὸν τοῖς πλείοσιν ὑπάρχον. εἰ δὲ μὴ εἴη ἀποδεικνύμενον διὰ τοῦ αἰτίου ἀλλὰ διὰ σημείου ἢ διὰ συμβεβηκότος μέσου λαμβανομένου, ἐνδέχεται ἄλλῷ δι' ἄλλου τὸ αὐτὸ ὑπάρχον δείκνυσθαι. (CAG 13.3, 593, 11–22)

Solving this and showing the duplicity of the cause (for the cause of the conclusion is not always the cause of the thing also) showing this <Aristotle> says that, if we have demonstrated *per se*, that is if the demonstration is produced through the cause, which is the cause of the thing, it is necessary that the same cause is assumed of all the things of which the same is demonstrated. Of this <Aristotle> provides the reason, already shown, saying <that> the definition of the extreme <term> which is the predicate and which belongs to many is <the cause> through which the demonstration is carried out, and this <i.e. the definition> is the middle term: in fact this cause belongs to many in a somehow general and identical way. If we could not demonstrate through the cause, but through a sign or assuming a middle that is an accident, <then> it is possible to demonstrate that the same thing belongs to one through the other. (our transl.)

The anonymous observes that 'being the cause of the conclusion' in a syllogism is not always the same as 'being the cause of the thing'; that is, it is not always the case that the middle term by which a syllogism is produced, and which thus may be considered *the cause of the drawing of the conclusion*, is also *the cause of the fact* stated in the conclusion (i.e., of the predicate of the conclusion or major extreme being predicated of the

¹ See also Ebbesen (2012: 363). It has to be noted that Moraux did not prove Alexandrian authorship for *everything* in the anonymous commentary. See the next footnote.

thing or minor extreme). When this happens, the anonymous says, we have not only a syllogism, but a proper or causal demonstration. In a proper or causal demonstration, the cause must be unique. The reason, it is suggested, is to be found in what Aristotle says at 99a3, namely that the middle is the definition of the major extreme. We have seen in the previous section that this was indeed Aristotle's argument for the uniqueness of the cause: the uniqueness of the definition entails the uniqueness of the cause. If the middle term is not the cause of the fact expressed in the conclusion, the demonstration is not a proper or causal demonstration, but only a demonstration 'through a sign or an accident', through which the cause is proved by means of the effect; and in this case, nothing guarantees that the cause will be unique.

The anonymous also offers a cursory interpretation of the difference between 'sign' and 'accident':

Τὸ μεν σημεῖον ἀεί τῷ πράγματι παρακολουθεῖ, τὸ δὲ συμβεβηκὸς δύναται και ἔξωθεν <είναι> (CAG 13.3, 593, 25–26)

The sign always accompanies the thing, the accident can also <be> from outside. (our transl.)

This seems to suggest that while a sign is causally produced by the thing, and thus 'accompanies' it, the accident is not causally connected with the thing, and is (or comes) 'from outside'. A $\sigma\eta\mu\epsilon$ iov in the wide sense of *APr*. II.27, 70b1, indeed, always follows from the cause—for either the cause is convertible with it, as in Aristotle's $\tau\epsilon\kappa\mu\eta\rho\iota$ ov (case I), or the cause is more extended than it (sub-case II.b), as in Aristotle's $\sigma\eta\mu\epsilon$ iov in the strict sense of *APr*. II.27, 70b4, in which latter case the effect still 'accompanies' the cause, even though the cause does not always follow from the effect. An accident, on the contrary, has neither of these relations with the thing in which it inheres.

In commenting on *APo*. II.17, 99b4, the anonymous addresses the question whether it is possible that there may be several causes of the same thing, and explicates Aristotle's answer with reference to the distinction, which he has just discussed, between 'being the cause of the conclusion' and 'being the cause of the thing'

τοῦτο δ' ἐπὶ μὲν τῶν ἐν συλλογισμῷ αἰτίων τοῦ συμπεράσματος, ἐφ' ὧν νῦν ποιεῖται τὸν λόγον, οἶόν τε: εἶπε γὰρ 'ἂν διὰ συμβεβηκότων καὶ σημείων ὁ συλλογισμὸς γένηται'. ἐπὶ

δὲ τῶν ἀποδείξεων καὶ τῶν δι' αἰτίου συλλογισμῶν οὐχ οἶόν τε ὃν ἐδείχθη· (CAG 13.3, 597, 25–28)

This regards the causes of the conclusion in the syllogism, of which <Aristotle> is now speaking. <He> says indeed 'if the syllogism is produced through accidents or signs'. That this is not possible as regards demonstrations and syllogisms through the cause has been shown. (our transl.)

When the 'cause' is taken in the sense of being the cause of the drawing of the conclusion in a syllogism, it is possible that there may be several causes of the same thing. The reference is again to the $\sigma\nu\lambda\lambda\rho\gamma\sigma\mu\delta\zeta$ $\deltai\delta$ $\sigma\nu\mu\beta\epsilon\beta\eta\kappa\delta\tau\omega\nu\kappa\alpha$ is $\sigma\eta\mu\epsilon\omega\nu$ mentioned at 99a3: in a syllogism through signs, the middle term is the cause of the drawing of the conclusion, but is not the cause of the fact expressed in the conclusion. When, by contrast, the 'cause' is taken in the sense of being the cause of the thing itself (i.e., of the fact expressed in the conclusion) it is not possible that the syllogism is produced by more than one cause. In this latter case, we have a demonstration proper or syllogism through the cause; in the former case, we only have a syllogism through signs.

If we accept that the commentary on the *Posterior Analytics* contained in the anonymous commentary in CAG 13 derives from Alexander's lost commentary, it is possible to conclude that the contrast between demonstration proper and demonstration through signs was actually taken up and commented upon by the Aphrodisiensis in connection with *APo*. II.17, 99a1–4 and 99b4 (the anonymous' commentary on the first book is missing). ¹ Further evidence of Alexander's discussion of this contrast comes from his commentaries on the *Prior Analytics* and on the *Topics*. Let us begin with the latter.

In commenting upon *Top.* 100a27–29, where Aristotle says that a demonstration is a syllogism from premises that are true and primary, or from premises which have in their turn been demonstrated from true and primary premises, Alexander explains that a primary premise is one that contains the cause of the conclusion, for 'what is primary gives the cause of what comes after' ($\tau \dot{\alpha} \gamma \dot{\alpha} \rho \pi \rho \tilde{\omega} \tau \alpha \tau \tilde{\omega} \nu \mu \epsilon \tau \dot{\alpha} \tau \tau \tilde{\omega} \tau \dot{\alpha} \tau \dot{\alpha} \delta \iota' \alpha i \tau (\omega \nu t)$). Thus, 'demonstration is the syllogism through the cause' ($\dot{\delta} \gamma \dot{\alpha} \rho \delta \iota' \alpha i \tau (\omega \nu t)$)

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¹ Besides Moraux's argument for the Alexandrian authorship of the commentary as a whole, the attribution of the scholia *ad* 99a1 and *ad* 99b4 to Alexander is independently justified by the fact that in them a distinction is used (that between 'being the cause of the conclusion' and 'being the cause of the thing') which we have seen is an authentically Alexandrian passage (cf. *In APr.* 21, 10–23).

συλλογισμός ἀπόδειξις, 16, 3–4). At this point the distinction between syllogism of the why and syllogism of the that is silently introduced by means of the eclipse example, which comes directly from APo. I.13, the official treatment of that distinction. The argument that shows that the moon is eclipsed because it is screened by the earth is a demonstration, while the argument that shows that the moon is screened by the earth because it is eclipsed is not a demonstration in the strict sense (οὐκέτι ἀποδείκνυσι κυρίως, 16, 13), because the cause is inferred from the effect. Alexander also offers the example of the lactating woman, which comes directly from APr. II.27, the official treatment of signinferences: the argument that shows that a woman is lactating because she has born a child is a demonstration in the strict sense, while the argument that shows that she has born a child because she is lactating is not a demonstration in the strict sense. A demonstration through the effect is not a demonstration in the strict sense ($\kappa u \rho i \omega c$), but only in a secondary sense (δευτέρως, 16, 29).

In his commentary on the *Prior Analytics* Alexander explicitly contrasts demonstration with the syllogism through signs. The context in which the contrast occurs is Aristotle's definition of the syllogism:

A syllogism is a discourse in which, certain things having been supposed, something different from the things supposed results of necessity inasmuch as they are the case ($\tau \tilde{\varphi}$ $\tau \alpha \tilde{\upsilon} \tau \alpha$ $\tilde{\upsilon} \tau \alpha$). By 'inasmuch as they are the case', I mean 'resulting through them' ($\delta i \alpha \tau \alpha \tau \sigma \upsilon \mu \beta \alpha i \upsilon \varepsilon \upsilon$), and by 'resulting through them' I mean 'needing no further term from outside in order for the necessity come about' (*APr.* I.1, 24b18–22; transl. Smith, modified)

Aristotle says that in a syllogism the conclusion results necessarily from the premises 'inasmuch as they are the case' ($\tau \tilde{\varphi} \tau \alpha \tilde{\upsilon} \tau \alpha \tilde{\upsilon} \iota \alpha \tilde{\upsilon} \alpha \tilde{\upsilon} \iota \alpha \tilde{\upsilon} \alpha \tilde{\upsilon} \iota \alpha \tilde{\upsilon} \alpha \tilde{\upsilon} \alpha \tilde{\upsilon} \iota \alpha \tilde{\upsilon} \alpha \tilde{\upsilon} \iota \alpha \tilde{\upsilon} \omega \tilde{\upsilon} \iota \alpha \tilde{\upsilon} \omega \tilde{\upsilon} \iota \alpha \tilde{\upsilon} \iota \alpha \tilde{\upsilon} \iota \alpha \tilde{\upsilon} \iota \alpha \tilde{\upsilon} \omega \tilde{\upsilon$

As to why he added 'inasmuch as they are the case' ($\tau \delta \tau \tilde{\omega} \tau \alpha \tilde{\upsilon} \tau \alpha \tilde{\upsilon} \tau \alpha \tilde{\upsilon} \tau \alpha$) to the definition of the syllogism, he himself explained this when he said: 'By "inasmuch as they are the case" I mean that 'it comes about because of them' ($\delta \iota \alpha \tau \alpha \tilde{\upsilon} \tau \alpha \sigma \upsilon \mu \beta \alpha (\upsilon \varepsilon \upsilon)$). This itself might still seem less than plain. For 'because of them' betokens an explanation, and yet there can be syllogisms which do not proceed by way of explanations—for example, syllogisms by way of signs ($\delta i \dot{\alpha} \sigma \eta \mu \epsilon i \omega \nu$) which prove what is primary from what is posterior. This feature—viz. being syllogized by way of explanations—is a proper characteristic of demonstrations. For although the premises must indeed be explanatory of the conclusion if there is to be a syllogism, what is meant by the premises need not always be explanatory of what is meant by the conclusion. (For you can also syllogize what is prior by way of what is posterior—proving that she has given birth from the fact that she is lactating, or that there was a fire from the ashes—and in general, syllogisms by way of signs (oi $\delta i \dot{\alpha} \sigma \eta \mu \epsilon i \omega \nu \sigma \upsilon \lambda \lambda \circ \eta \upsilon \sigma \upsilon \mu \delta i$) are of this sort: for the posterior is not explanatory of the prior). This is why he also explained 'it comes about because of them' ($\delta i \dot{\alpha} \tau \alpha \overline{\upsilon} \upsilon \beta \alpha i \omega \omega$), by saying that they 'need no external term for the generation of the necessity', i.e. that the terms laid down are sufficient in themselves for the conclusion. (Alexander, *In APr.* 21, 10–23, transl. Barnes et al. pp. 71–72)

In a general sense, in any syllogism of whatever kind the conclusion comes about 'because of' the premises. In the *Metaphysics* Aristotle says that the premises are 'causes' (αἴτιά) of the conclusion in the sense of 'that from which' ($\tau \delta \ \epsilon \xi \ o \ \delta$) (1013b21) the conclusion is obtained. In this general sense, the premises are better characterized as the cause of the drawing of the conclusion, but not of the conclusion itself (the premises 'must indeed be explanatory of the conclusion if there is to be a syllogism'). Strictly speaking, however, only in that specific kind of syllogism which is a demonstration are the premises 'cause' of the conclusion in the sense that they are the causes of the conclusion itself, i.e., of the fact expressed in the conclusion (the premises 'need not always be explanatory of what is meant by the conclusion'). Without some such distinction, Alexander suggests, it would be impossible to explain how a syllogism of the that or through signs, in which the premises are not 'causes' in the strict sense of the conclusion, could qualify as a syllogism at all. His examples are the lactating woman from APr. II.27, which is a sign of her having born a child, and the ashes, which are a sign of there having been a fire. As Alexander clearly sees, Aristotle's explanation at 24b21–22 is intended precisely not to exclude non-demonstrative syllogisms from his definition of the syllogism. For in explaining that διὰ ταῦτα συμβαίνειν means that the conclusion results through those premises *alone* he is clearly implying that in any syllogism whatever (and thus also in syllogisms of the that or through signs) the conclusion follows with necessity from appropriate premises and 'because of them', provided that 'because of them' is taken in the general sense of being the cause of the drawing of the conclusion,

and not in the strict sense of being the cause of the fact expressed in the conclusion. $^{\rm 1}$

We see that the distinction between the two senses in which the premises are 'causes' of the conclusion which Alexander discusses in his commentary on the Prior Analytics precisely corresponds to the distinction between 'being the cause of the conclusion' and 'being the cause of the thing' that we have encountered in the anonymous commentary: in any syllogism whatever the conclusion follows from the premises and 'because of them', so that the premises may be said to be the cause of the conclusion; but only in a proper demonstration do the premises contain the cause of the thing, i.e., the cause of the fact expressed in the conclusion. As the anonymous puts it in the commentary on APo. II.17 99a1-4, 'the cause of the conclusion is not always the cause of the thing also' (CAG 13.3, 593, 12-13), i.e. (in the terms of Alexander's commentary on APr. I.1, 24b18-22) 'what is meant by the premises need not always be explanatory of what is meant by the conclusion' (In APr. 21, 18). Syllogisms through signs are precisely those syllogisms in which the premises are 'causes' of the conclusion in the general sense of being the cause of the drawing of the conclusion; but they do not contain the 'cause of the thing', and thus are not causes of the fact expressed in the conclusion.

In his commentary on the Topics, Alexander identified both the inference about the lactating woman of APr. II.27, which is an irrefutable or deductively valid sign-inference (τεκμήριον), and the inference about the eclipse of APo. I.13, which is a syllogism of the that, with the demonstration in a secondary sense ($\delta \epsilon \upsilon \tau \epsilon \rho \omega c$). He has therefore *implicitly* identified the deductively valid sign-inference or τεκμήριον of APr. II.27 with the syllogism of the that of APo. I.13. In the passage from the commentary on the Prior Analytics, he has explicitly identified the syllogism through signs in general (οί διὰ σημείων συλλογισμοί) with the argument by which we prove what is primary from what is posterior, which is again a clear reference to the syllogism of the that of APo. I.13. The examples of syllogism through signs provided in this latter context are again that of the lactating woman from APr. II.27 and that of the presence of ashes as sign of a past (and extinguished) fire. Aristotle never explicitly says that a syllogism of the that can be considered as a syllogism through signs, even though in the two parallel passages in APo. I.6 and II.17 he comes very near to imply some such identification. Alexander explicitly

¹ Cf. Gili (2011: 103).

identifies the two varieties of syllogism, and by so doing inaugurates an exegetical pattern that would become quite popular among commentators on the *Posterior Analytics*.

3. Themistius

The paraphrase of Themistius (c. 317–390 CE) is the oldest surviving companion to the whole of the *Posterior Analytics*. The first reference to sign-inferences to be found in this work is in the discussion of *APo*. I.2, where Aristotle defines the scientific syllogism or demonstration. Themistius compares the scientific syllogism or demonstration with unscientific syllogisms:

ό γὰρ ἐπιστημονικὸς συλλογισμὸς τούτῷ μάλιστα τῶν λοιπῶν διενήνοχεν. ἐν μὲν γὰρ τοῖς ἄλλοις δείκνυται καὶ διὰ ψευδῶν τὸ ἀληθές, ὡς παρὰ τοῖς ῥήτορσι πολλάκις, καὶ δι' ὑστέρων τὸ πρότερον, ὥσπερ οἱ διὰ σημείων συλλογισμοί, καὶ δι' ἀληθῶν μὲν οὐκ οἰκείων δέ, ὥσπερ εἴ τις ἰατρὸς τὰ περιφερῆ τῶν τραυμάτων δυσιατότερα ἀποδεικνύοι, διότι τὸ σχῆμα πολυχωρητότερον τῶν λοιπῶν. γεωμέτρου γὰρ ἡ ἀπόδειξις, οὐκ ἰατροῦ. (CAG 5.1, 6, 20–26)

The scientific syllogism differs above all from the remaining <kinds> of <syllogism> in this respect. For in the other cases the true is shown through the false, as often by rhetoricians, and that which is prior <is shown> through that which is posterior, as in the syllogisms through signs, and through premises that are> true, but just not appropriate <for the discipline>, as if the physician were to prove that the circular wound is the most difficult to heal because this figure contains a larger area than the others. For this demonstration is that of a geometer, not of a physician. (our transl.)

At *APo*. I.2, 71b21–22 Aristotle had explained that in order for a syllogism to qualify as a demonstration its premises must be prior to the conclusion. Themistius observes that when the premises are posterior to their conclusion, that which is prior is shown through that which is posterior, and this is a syllogism through signs. For a sign is something posterior from which we know something which is prior.

Themistius' most exhaustive discussion of sign inferences occurs however in the context of his commentary on *APo*. I.13, the official presentation of the distinction between the syllogism of the why and the syllogism of the that. We have seen that according to *APo*. I.13 the syllogism of the that differs from the syllogism of the why both across sciences and within the same science, and that within the same science there is again a twofold distinction, according to whether the syllogism does not proceed from immediate premises (78a23–26) or whether it does proceed from immediate premises but infers the cause from the effect (78a26–29). In commenting upon this second case, Themistius does something that Aristotle does not: he associates the syllogism of the that with the syllogism through signs:

ἕτερος δὲ τρόπος, ὅταν δι' ἀμέσων μὲν ἀμφω, ἀλλ' ὁ μὲν διὰ τῆς αἰτίας ὁ δὲ διὰ τοῦ σημείου. ὁ μὲν γὰρ διὰ τοῦ τετοκέναι τό γάλα ἔχειν ἀποδείκνυσι τὸ διότι, ὁ δὲ διὰ τοῦ γάλα ἔχειν τό τετοκέναι τὸ ὅτι. καὶ ὁ μὲν τὰς αὐξήσεις τῆς σελήνης διὰ τοῦ σφαιροειδοῦς τὸ διότι, ὁ δὲ τὸ σφαιροειδὲς διὰ τῶν αὐξήσεων τὸ ὅτι. (CAG 5.1, 28, 15–19)

In another way <knowledge that and knowledge why differ> when both are through immediates, but one is through the cause and the other through the sign. For the one <that proves> the possession of milk through having given birth proves the why, while the one <that proves> having given birth through the possession of milk proves> the that; and the one <that proves> the waxing of the moon through its being spherical proves> the why, while the one <that proves> its being spherical through its waxing proves> the that. (our transl.)

The example of the waxing of the moon, as that which allows one to infer its being spherical, comes from *APo*. I.13, and is there characterized by Aristotle as a syllogism of the that. The example of the lactating woman, by contrast, comes from *APr*. II.27, and is there characterized by Aristotle as an irrefutable or deductively valid sign-argument or an argument based on a $\tau \epsilon \kappa \mu \eta \rho \iota ov$. Since Alexander's commentary on the *Prior Analytics* was presumably available to Themistius, it is no hazardous speculation to maintain that Alexander is the source for the association of the $\tau \epsilon \kappa \mu \eta \rho \iota ov$ of *APr*. II.27 with the demonstration of the that of *APo*. I.13 that Themistius makes when commenting on this passage.¹

¹ Although Alexander is cited only once in Themistius' paraphrase (at 20, 15; cf. Moraux 1979: 4), its possible dependence on Alexander's lost commentary is also suggested by the following circumstance. At 5.20–24 Themistius distinguishes the case in which the premises in a syllogism are causes of the conclusion from the case in which they are the cause of the thing demonstrated in the conclusion. His example is the inference of the presence of fire from the presence of ashes: the presence of ashes is the cause of the conclusion, but it is not the cause of the thing stated in the conclusion. As we know, this is precisely Alexander's distinction between being the cause of the drawing of the conclusion and being the cause of the fact stated in the conclusion, which we find both in his commentary on the *Prior Analytics* (ad 24b21–22, where the fire/ashes example is used) and in the anonymous commentary containing excerpts from Alexander (cf. above, §2). The dependence of Themistius' paraphrase on Alexander is conjectured on the basis of the same texts (except the anonymous) by Borgo (2009: 188–192).

Themistius continues by considering the two cases dealt with by Aristotle at *APo*. I.13, namely those which we referred to above as case I (cause and effect convert) and case II (cause and effect do not convert). The examples adduced in the previous passage (the waxing of the moon, the lactating woman) clearly belong to case I. The inference of the sphericity of the moon from its phases is a syllogism of the that or through signs, while the inference of the phases from its sphericity is a syllogism of the why or through the cause of the thing; likewise, the inference of a woman's having given birth through her lactating is a syllogism of the that or through signs, while the inference of her lactating is a syllogism of the that or through signs of the why or through the cause of the there are solved as a syllogism of the that or through signs, while the inference of her lactating from her having given birth is a syllogism of the why or through the cause of the there are solved as a syllogism of the that or through signs, while the inference of her lactating from her having given birth is a syllogism of the why or through the cause of the thing:

πολλάκις μὲν οὖν συμβαίνει καὶ ἀντιστρέφειν ἀλλήλοις τὸ αἴτιον καὶ τὸ σημεῖον καὶ ἄμφω δείκνυσθαι δι' ἀλλήλων, διὰ τοῦ σημείου μὲν ὡς τὸ ὅτι, διὰ θατέρου δὲ ὡς τὸ διότι, δι' οὖ μὲν ὡς γνωριμωτέρου τοῦ συμπεράσματος, δι' οὖ δὲ ὡς αἰτίου τοῦ πράγματος. (CAG 5.1, 28, 19–23)

Often it certainly happens that cause and sign convert with one another and either is proved through the other, the that through the sign, the why through the other; through the former when $\langle it is \rangle$ more known than the conclusion, through the latter when $\langle it is \rangle$ the cause of the thing. (our transl.)

But cause and sign do not always convert:

πολλάκις δὲ οὐκ ἀντιστρέφει τὰ σημεῖα τοῖς αἰτίοις αὐτών. εἰ μὲν γὰρ καπνός, πάντως καὶ πῦρ· εἰ δὲ πῦρ, οὐ πάντως καπνός⁻ καὶ εἰ μὲν τέτοκεν, ἀνδρὶ πεπλησίακεν⁻ εἰ δὲ πεπλησίακεν, οὐ πάντως τέτοκεν. ἐπὶ δὴ τῶν τοιούτων ἡ μὲν τοῦ ὅτι δεῖξις ἔστιν, ἡ δὲ τοῦ διότι ἐκλείπει. διὰ μὲν γὰρ τοῦ σημείου τὸ αἴτιον ἔστιν ἀποδεῖξαι, διὰ δὲ τοῦ αἰτίου τὸ σημεῖον οὐκέτι. (CAG 5.1, 28, 23–28)

Often the signs and the causes of the same things do not convert. If <there is> smoke, <there is> inevitably fire. If <there is> fire, <there is> not inevitably smoke. And if <a woman> has given birth, <she> has had intercourse with a man. If <she> has had intercourse with a man, it is not inevitable that she has given birth. In such cases there is demonstration of the that, while the <demonstration> of the why is missing, for it is possible to demonstrate the cause through the sign, but not <to demonstrate> the sign through the cause. (our transl.)

Having given birth and having had intercourse with a man do not convert, the intercourse is the cause of giving birth, but not all occurrences of the cause are accompanied by an occurrence of the effect. Themistius has in mind what we have referred to above as sub-case II.a, in which the cause has wider extension than the effect (all occurrences of the effect are occurrences of the cause, but not all occurrences of the cause are occurrences of the effect). If from the fact that a woman has given birth we infer that she has had intercourse with a man, the syllogism is of the that, because the cause (intercourse with a man) is inferred from a sign or effect of it (giving birth). But there cannot be a syllogism of the why, because the cause is more extended than the effect (not all intercourses have that effect).

Therefore, when cause and effect/sign convert (case I), there can be a syllogism of the why (the effect/sign is inferred from the cause) and a syllogism of the that (the cause is inferred from the effect/sign); this latter is called by Themistius $\delta i a \tau o \tilde{v} \sigma \eta \mu \epsilon i o v a \pi \delta \delta \epsilon i \xi \varsigma$. When cause and effect do not convert (case II), and the cause has wider extension than the effect (sub-case II.a), the syllogism can only be of the that, but cannot be of the why. Since in sub-case II.a all occurrences of the effect are occurrences of the cause, the demonstration that a less known cause occurs because one of its better known effects or signs occurs is irrefutable, and indeed a deductively valid syllogism. According to *APr*. II.27, such an inference qualifies as a first-figure sign or $\tau \epsilon \kappa \mu \eta \rho t o v$.

Themistius also considers the case of converting effects:

ώσπερ οὖν οὐδὲ πάντα τὰ αἴτιά τε καὶ αἰτιατὰ ἀντιστρέφει, οὕτως οὐδὲ πάντα τὰ ἀντιστρέφοντα αἴτιά τέ ἐστι καὶ αἰτιατά. δυνατὸν γὰρ τοῦ αὐτοῦ αἰτίου πλείω σημεῖα πρὸς ἄλληλα ἀντιστρέφειν, οἶον τοῦ πυρέττειν σημεῖα ἥ τε ταραχὴ τῆς ἀρτηρίας καὶ ἡ τοιάδε θερμότης. δείκνυται μὲν οὖν δι' ἀλλήλων καὶ τὰ τοιαῦτα, κατ' οὐδέτερον δὲ ὁ τοῦ διότι συλλογισμός, ἀλλὰ κατ' ἀμφότερα ὁ τοῦ ὅτι. (CAG 5.1, 28, 28–29, 3)

And just as not all causes and effects convert, so not all terms that convert are causes and effects. It is indeed possible that several signs of one and the same cause convert with one another, for example the signs of fever which <are> both tracheitis and this kind of heat. While these things indicate each other, according to neither is there a syllogism of the why but according to both <there is a syllogism> of the that (our transl.)

It may happen that two or more effects follow from one and the same cause, and that these effects convert with one another. The example offered is that of the infection of the trachea and the heat of the body, which are both effects of fever and which convert (all cases of tracheitis are cases of heat of the body, and vice versa). Both the tracheitis and corporal heat are effects, and thus signs, of fever. Inferring the fever from either of its effects or signs counts as a valid syllogism of the that.¹

Are converting effects also signs of each other? Themistius says that either of two (or more) converting effects δείκνυται the other, not that either is a $\sigma\eta\mu\epsilon$ iov of the other. However, a case can be made for the view that converting effects 'indicate' in the sense of being each the sign of the other. For that which enables them to function as signs at all-their converting with the cause while being more known than it—is also what enables either to function as a sign of any co-extensive effect, or in other words of another sign of the same thing. Both tracheitis and corporeal heat are better known than their cause (fever), and thus they are able to function as signs of it. But it may also be the case that in certain circumstances either is more known than the other: we may be able to measure corporeal heat but unable to detect tracheitis. In this case, since the effects convert, one can be said to indicate or be a sign of the other. We would thus have a syllogism of the that or τεκμήριον from either effect to the cause when the converting effects are equally known, and from either effect to the other effect when the former is more known than the latter.

Be that as it may, Themistius is the only commentator on the *Posterior Analytics* to have explicitly considered the inferential behaviour of the effects of the same cause. The topic seems to have escaped the attention of later interpreters.

4. Philoponus

In his commentary on the first ² book of the *Posterior Analytics*, Philoponus (ca. 490–570 CE) declares himself to be reporting the teachings of his master Ammonius (440–523 CE). While Alexander is

¹ The example is not Aristotelian. In the *Sophistical Refutations* the inference from a man's being hot to his being in a fever is an instance of the fallacy of the consequent (167b20), and accordingly a refutable sign-inference. In the *Rhetoric* fever is a necessary sign (τεκμήριον) of illness (1357b15–16), and hard breath a refutable sign (σημεῖον in the strict sense) of fever (1357b18–21). In the first example the fever is the sign, in the other the cause revealed by the sign. Themistius' example may derive from medical literature. The expression ταραχή τῆς ἀρτηρίας is here rendered with 'tracheitis', a bacterial infection of the windpipe resulting in severe cough, breathing difficulty and usually going together with high temperature.

² The commentary to the second book published in CAG 13.3 was deemed to be spurious already by Wallies, cf. CAG 13.3, v–vi; cf. also Ebbesen (2012: 363), and, for a different take on the attribution, Goldin (2009: 1–4).

criticized at various places in Philoponus' commentary, ¹ it is highly probable that he never had Alexander's commentary in his hands, and that in criticizing Alexander he was in fact relying on Ammonius.² The same could apply to another *prima facie* obvious source for Philoponus, namely Themistius' paraphrase; ³ but the fact that on at least one occasion Philoponus invokes Themistius against Ammonius' interpretation of *APo*. I.13, 78b28–34 might suggest that he knew the former's work independently of the latter.⁴

In commenting upon I.2, 71b22, where as we have seen Aristotle says that the premises of a demonstration must be prior to and causes of the conclusion, Philoponus observes that 'if we go backwards from the effect to the cause, such a thing is no longer a demonstration but a sign (τεκμήριον). For example, "the sun is eclipsed, that which is eclipsed is screened, therefore the sun is screened".' (CAG 13.3, 26, 14-15). The paradigmatic example of a syllogism of the that of I.13 is here considered not as a demonstration but as a sign. It is important to note that the term used by Philoponus is $\tau \epsilon \kappa \mu \eta \rho i ov$, not $\sigma \eta \mu \epsilon i ov$. The reference is plainly to the distinction between deductively valid and deductively invalid signinferences outlined by Aristotle at APr. II.27. Although as we have seen (§§2-3) already Alexander and Themistius had given a semiotic characterization of the syllogism of the that of APo. II.13, Philoponus is the first to perceive that a syllogism of the that, though not causal, is a deductively valid syllogism, and thus merits the status of irrefutable sign or $\tau \epsilon \kappa \mu \eta \rho to v.^5$

According to Philoponus (on 72a7), a τεκμήριον is a demonstration, though only in a secondary sense:

demonstration in the strict sense ($\kappa \nu \rho (\omega \varsigma \dot{\alpha} \pi \delta \delta \epsilon \xi \iota \varsigma)$ in fact should confirm things that are secondary and less clear on the basis of things that are primary, immediate, and better known. But since what is better known in nature is not in all cases better known to us too, it often happens that we construct our proofs of things that are prior on the basis of things that are posterior, on account of the fact, as I said, that the things that are prior are not

¹ CAG 13.3, 4, 1; 41, 1; 62, 2, 13–14; 111, 20–32; 122, 11; 126, 4; 139, 9; 159, 18; 160, 9–14; 174, 5–9; 181, 11; 196, 9.

² Cf. Moraux (1979: 5); McKirahan (2008: 2).

³ Cited at CAG 13.3, 48, 7; 70, 7; 138, 6; 177, 28.

⁴ CAG 13.3, 177, 19–178, 13.

⁵ Philoponus also uses τεκμήριον in the authentically Aristotelian sense of irrefutable sign in his commentary on Aristotle's *Meteorologica*; cf. CAG 14.1, 40, 15; 47, 20; 49, 29; 99, 16; 100, 5; 102, 34; 110, 34; 111, 21; 118, 22.

better known to us. And this kind of proof is called <a proof> from a sign (τεκμηριῶδης) and irrefutable (ἄλυτον, 31,11). As a result it has received the name demonstration as well. For demonstration in the strict sense, as I said, is one that confirms things that are secondary on the basis of things that are prior, when being primary and known in nature and being better known to us coincide. But when this does not obtain, but we are compelled to confirm things that are prior on the basis of things that are posterior, this kind of proof is called <a proof> from a sign (τεκμηριῶδης), and because of the irrefutability of signs (διὰ τὸ ἄλυτον τῶν τεκμηρίων), it has been deemed worthy of the name 'demonstration'. (CAG 13.3, 31, 6–17; transl. McKirahan, p. 41)

We have seen (§2) that already Alexander, in his commentary on the *Topics*, had distinguished demonstration in the strict sense (κυρίως), which is one from the cause to the effect, from demonstration in a secondary sense (δευτέρως), which is one from the effect to the cause. Philoponus echoes Alexander's distinction when he says that 'since such indicators or signs (σημεῖα ἢ τεκμήρια) are irrefutable (ἄλυτα), this is why we call proofs based on them demonstrations, according to a secondary standard of demonstration (κατὰ δεύτερα μέτρα ἀποδειξεως)' (CAG 13.3, 32, 5–7; transl. McKirahan, p. 42). A 'tekmeriodic demonstration' is not a demonstration in the strict sense; but since it is irrefutable (deductively valid), it deserves the title of demonstration, though only κατὰ δεύτερα μέτρα.¹

Philoponus' use of the disjunction σημεῖα ἢ τεκμήρια at 32, 6 should not mislead us into thinking that he overlooks the syllogistic distinction between σημεῖα and τεκμήρια. The following passage (on 72b25–8) is clear evidence that he was sensitive to that distinction:

establishing prior things through posterior things is not always necessary, except in cases where the posterior things are irrefutable signs (ἄλυτα τεκμήρια, 49, 5–6), such as 'since there is ash, fire was once here', or 'since the moon is illuminated in this way, it is spherical'. However, if from the fact that a woman is pale it is established that she has given birth, since the indicator is refutable (λυτοῦ σημείου, 49, 9–10) such <an argument> would not be called a demonstration in any way. And in the case of irrefutable signs (ἀλύτων τεκμηρίων, 49, 11) we will not say that such <a demonstration> is demonstration in the strict sense, but that this whole thing is a demonstration from a sign (τεκμηριώδη ἀπόδειξιν, 49, 12), since it is necessary to establish effects from their causes, which is a

¹ Cf. the following passage (on 72b31-2): 'if the definition of demonstration has been given well by us, that posterior things must be established on the basis of things prior in nature, a <demonstration> that establishes prior things on the basis of posterior things will not be called demonstration in the strict sense, but, as we said, <it will be called> a proof from a sign (τεκμηριώδης δεῖξις)' (CAG 13.3, 50, 5–9; transl. McKirahan, p. 57).

property of demonstration in the strict sense, and not causes from their effects. (CAG 13.3, 49, 5–14; transl. McKirahan, p. 56)

Fire and ash are cause and effect that convert (whenever there is fire there is ash, and whenever there is ash there was a fire). The syllogism that infers the effect from the cause is a syllogism of the why, and qualifies as a demonstration proper; the syllogism that infers the cause from the effect is a syllogism of the that or through signs. But since cause and effect reciprocate, this syllogism is irrefutable and deductively valid, and thus is, properly speaking, a syllogism from a $\tau \epsilon \kappa \mu \eta \rho \iota ov$. Its character of irrefutability, Philoponus says, allows us to call it an $\dot{\alpha}\pi \delta \delta \epsilon \iota \xi \iota \zeta$, though only $\kappa \alpha \tau \dot{\alpha} \delta \epsilon \dot{\upsilon} \tau \epsilon \rho \alpha$ µ $\dot{\epsilon} \tau \rho \alpha$; it is therefore also called a $\tau \epsilon \kappa \mu \eta \rho \iota \delta \delta \iota \xi \iota \zeta$. The same applies to the inference of the sphericity of the moon from its phases (one of the examples of syllogism of the that of *APo*. I.13).

By contrast, pregnancy and paleness are cause and effect that do not convert (a pregnant woman is pale, but not all pale women are pregnant). The syllogism that infers the effect from the cause is a syllogism of the why, and therefore should qualify as a demonstration proper; but the syllogism that infers the cause from the effect is *not* a syllogism of the that, and thus is no demonstration at all, either in its primary or in its secondary sense. Rather, it is an inference from a σημεῖον, and here σημεῖον has to be taken in the strict or marked sense recorded at APr. II.27, 70b4, i.e., as a deductively invalid or refutable sign (λύσιμον σημεῖον). In point of fact, the example chosen by Philoponus (pale woman) comes directly from APr. II.27, where it instantiates a second-figure sign-inference. The case of nonconverting cause and effect that Philoponus calls a $\sigma\eta\mu\epsilon\tilde{i}\sigma\nu$ is what we have referred to above as sub-case II.b. in which the effect has wider extension than the cause (all occurrences of the cause are occurrences of the effect, but not all occurrences of the effect are occurrences of the cause) and as such it only admits of a demonstration why.

This picture is confirmed by Philoponus' comments on I.6, 75a31– 3, the first of the two passages of the *Posterior Analytics* in which Aristotle himself contrasts demonstration proper with sign-inferences. He says:

Because everything that does not belong *per se* belongs accidentally and things that belong accidentally can also not belong, therefore demonstration cannot be based on such things. For even if such accidents are never separated from their subjects, he says, unless they belong to them *per se* there will not be a demonstration based on them, for reasons that have been stated many times. As an example of this he gives syllogisms through signs (δ ià σημείων συλλογισμούς), which infer causes from effects. For it is from the

moon's phases that we infer that it has a spherical body, and from smoke's appearing <that we infer> that there is a fire. (CAG 13.3, 97, 20–27; transl. McKirahan, p. 102, modified)

Again, the fact that here Philoponus calls the syllogism from the effect to the cause a $\delta i \dot{\alpha} \sigma \eta \mu \epsilon (\omega v \sigma \upsilon \lambda \lambda \delta \gamma i \sigma \mu \delta \zeta$ should not mislead us into thinking that he is ignoring the syllogistic distinction between $\sigma \eta \mu \epsilon \tilde{i} \alpha$ in the strict sense and $\tau \epsilon \kappa \mu \eta \rho i \alpha$. The examples adduced clearly indicate that he has in mind syllogisms of the that that are deductively valid and thus qualify as $\tau \epsilon \kappa \mu \eta \rho i \omega \delta \epsilon i \zeta \epsilon i \zeta$. His use of $\sigma \eta \mu \epsilon \tilde{i} \circ v$ in this passage has to be explained by the fact that he is indirectly *reporting* Aristotle's words. Once Aristotle's use of $\sigma \eta \mu \epsilon \tilde{i} \circ v$ instead of $\tau \epsilon \kappa \mu \eta \rho i \circ v$ is emended both conceptually and terminologically, there is no risk in explaining Aristotle's words by reporting them, Philoponus must have thought. An even more explicit instance of a report of Aristotle's (and his commentators') wording occurs in the passage to be presently considered.

Philoponus' most systematic discussion of sign-inferences is in his commentary on *APo*. I.13. Again, the relevant portion of Aristotle's text is 78a26–78b13, where the Stagirite investigates how syllogism of the that and syllogism of the why differ within the same science when both proceed from immediate premises but the one infers the cause from its effect and the other the effect from its cause. It should be recalled that Aristotle considers two cases: case I, in which cause and effect convert (78a26–78b10), and sub-case II.a, in which the cause has wider extension than the effect, so that the cause is inferable from the effect but not the effect from the cause (78b11–13). Philoponus first considers case I.

Some causes and effects reciprocate and others do not. For example, if there is fire there must be ash as well, and if there is ash, there must be fire as well. Also in the case of the phases of the moon; if it is illuminated in the way it appears, it must be spherical too, and if it is spherical it must be illuminated in that way. Now in cases where cause and effect reciprocate with one another, we frequently establish the cause on the basis of the effect because the effect is better known than the cause. For example, in proving that the moon is spherical on the basis of its phases—although the phases are not the cause of its being spherical, but rather that fact <that it is spherical> is <the cause> of them <i.e., the phases>. So this is called a syllogism of the 'that', since the sphericity of the moon is syllogized from its phases. Likewise 'ash is here, where there is ash there was fire, therefore there was fire here'. But if we were to say 'fire is here, where there is fire there must be ash as well, therefore there is ash here', the syllogism is of the 'that', since the effect is syllogized from the cause. The former is <a syllogism> of the 'that', since the cause is syllogized from the effect. And since such things are irrefutable (ǎ $\lambda \omega \tau \alpha$), they are called signs ($\sigma\eta\mu\epsilon$ ia $\lambda \epsilon\gamma\epsilon\tau\alpha$) and this is a secondary kind of demonstration ($\delta\epsilon\omega\tau\epsilon\rho\alpha$

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ἀπόδειξις), or as a whole it is a demonstration from a sign (τεκμηριώδης ἀπόδειξις). This holds for things that reciprocate. (CAG 13.3, 168, 24–169, 8; transl. McKirahan, p. 67, modified)

Fire and ash, as well as the sphericity of the moon and its phases, are cause and effect that convert. The inference of the cause from the effect is a syllogism of the that or from a sign. Since the convertibility of cause and effect guarantees the deductive validity of the inference, the effect is a τεκμήριον of its cause and the whole is a τεκμηριώδης ἀπόδειξις. We think Philoponus' use of σημεῖα at 168, 7 must again be taken as a report of Aristotle's and his commentators' wording: the premises of such inferences 'are called' σημεῖα, but since these inferences are irrefutable, their premises are more precisely to be called τεκμήρια. As already remarked, just like Alexander in his commentary on the *Topics* (see above, \S 2), Philoponus considers the syllogism of the that or τεκμηριώδης ἀπόδειξις not as a demonstration in the strict sense (κυρίως ἀπόδειξις), but as a demonstration in a secondary sense (δεύτερα ἀπόδειξις).

Philoponus next considers case II and both its sub-cases II.a and II.b.

But it often happens that when the cause occurs it is necessary for the effect to occur too, but not that when the effect occurs the cause must occur as well, and vice versa, that when the effect occurs the cause must too, but not <that when> the cause <occurs> the effect must too. An example of the former: if a women has given birth she must be pale, but the fact of having given birth does not always follow from the fact of being pale, for there can be several causes of the same thing: fear, illness and other things. Likewise if someone has just walked a lot, he is tired, but it is not the case that if someone is tired he has also just walked a lot. For it is possible to be tired from doing a lot of work. And clearly in these cases the syllogism will be of the 'why' and not just of the 'that', since the syllogism always proceeds through the causes establishing the effect on the basis of them. But it is not possible to prove the cause from the effect since they do not reciprocate. $[...]^1$ An example of the latter, namely where the cause is a consequence of the effect but the effect is not necessarily <a consequence> of the cause as well: if a woman has given birth, she has had intercourse with a man. But this cannot reciprocate—that if a woman has had intercourse with a man she has also given birth. Also, if there are fruits, there must have been rain, but if there has been rain there will not necessarily be fruits as well. In these cases the syllogism is only of the 'that' and never of the 'why'. (CAG 13.3, 169, 9-27; transl. McKirahan, pp. 67-8, modified)

¹ Here we omit the sentence 'And so in these cases the syllogism is of the 'that' and not at all of the 'why'', because it plainly contradicts the whole passage. Either the text must be emended or it must be considered as a later (and mistaken) interpolation.

Philoponus first considers sub-case II.b ('when the cause occurs it is necessary for the effect to occur too, but not that when the effect occurs the cause must occur as well', i.e., the effect has a wider extension than its cause). A woman who has given birth is pale, but not all pale women have given birth. We have seen above that at 49, 9–10 Philoponus explicitly maintains that inferring a woman's having given birth from her being pale (Aristotle's own example of second-figure sign-inference of *APr*. II.27) is an argument from a refutable sign or $\sigma\eta\mu\epsilon$ iov in the strict sense of *APr*. II.27, 70b4, and not at all a demonstration. Here, however, no allusion is made to the semiotic nature of such a syllogism. Philoponus limits himself to say that with II.b sub-cases the syllogism can only be of the why, and never of the that (for in fact, the syllogism of the that that results is a deductively invalid argument, and thus no syllogism of the that at all).

The converse case, sub-case II.a ('when the effect occurs the cause must too, but not <that when> the cause <occurs> the effect must too', i.e., the cause has a wider extension than its effect) is considered in turn. One of the examples chosen by Philoponus for this case is the one discussed by Themistius at 28, 23–28, and might come directly from him. A woman who has given birth has had intercourse with a man, but not all intercourses with men cause women to give birth. Note that here having given birth is the effect (of having had intercourse with a man), and not, as in the example of the pale woman from *APr*. II.27, the cause (of paleness). Themistius had said that in such cases there is demonstration of the cause through the sign, but not demonstration of the sign through the cause. In fact, as Philoponus explains, in sub-case II.a the syllogism can only be of the that (from effect to cause) and never of the why (from cause to effect). But again, Philoponus is silent about the semiotic nature of such inferences.¹

¹ According to Morrison (1997), Philoponus' doctrine of tekmeriodic proof differs from Aristotle in two ways: first, because unlike Aristotle, Philoponus considers induction as a kind of tekmeriodic proof; secondly, because unlike Aristotle, Philoponus does not recognize that some sign-inferences are not deductively valid but merely probable. We find both these claims to be inaccurate. As to induction, nowhere does Philoponus associate induction to sign-inferences. He says that induction has to do with the perception of particulars (17, 13; 18, 10–11; 214, 17; 215, 13; 216, 2–3, 13–14), that by induction we establish universals on the basis of particulars, i.e., prior things on the basis of posterior things (49, 20–22), and that by induction we learn axioms and postulates (215, 6–7). But these claims entail in no obvious way that induction is a kind of sign-inferences are, for Aristotle and his commentators, inferences from what

Since it is no demonstration at all, Philoponus should not, and in fact does not, call the syllogism from a refutable sign in the strict sense a 'semiotic demonstration' (just as the syllogism from a necessary sign is called by him a 'tekmeriodic demonstration'). But someone else did. In the commentary on the *Sophistical Refutations* by Michael of Ephesus (pseudo-Alexander),¹ the inferences associated with the fallacy of the consequent are called σημειώδεις ἀποδείξεις:

ού μὴν ἀλλὰ καὶ ἐν τοῖς ῥητορικοῖς συλλογισμοῖς αἱ σημειώδεις ἀποδείξεις ἐκ τῶν ἑπομένων γίνονται. ἐπειδὴ γὰρ σημεῖον τοῦ μοιχοῦ τὸ καλλωπίζεσθαι ἐστι καὶ τοῦ κλέπτου τὸ νυκτὸς πλανᾶσθαι (ἕπεται γὰρ καὶ τῷ μοιχῷ τὸ καλλωπίζεσθαι καὶ τῷ κλέπτῃ τὸ νυκτοπορεῖν), διὰ τοῦτο αἱ ῥητορικαὶ ἀποδείξεις ἐκ τῶν ἐπομένων γίνονται· ὁ γὰρ βουλόμενος δεῖξαι ὅτι μοιχός ἐστι, τὸ ἑπόμενον ἕλαβεν, ὅτι καλλωπιστής, καὶ ὁ τὸν κλέπτην ἐλέγξαι βουλόμενος ὅτι νύκωρ πλανᾶται. τὸ δὲ πολλοῖς μὲν ταῦτα ὑπάρχει, τὸ δὲ κατηγορούμενον οὐχ ὑπάρχει λύσις ἐστί. κατηγορούμενον δὲ λέγει τὴν μὲν μοιχείαν κατηγορουμένων (εἴρηται γὰρ ὡς οὐκ ἀνάγκῃ ἀντιστρέφειν), ἀλλ' ὡς ὑπὸ τῶν σοφιστῶν ὡς κατηγορουμένων λαμβανομένων. (ps-Alexander, *In SE*, CAG 2.3, 48, 27–49, 3)

Moreover, also in rhetorical syllogisms the semiotic demonstrations are produced from the consequences. For fancy dressing is a sign of the adulterer and wandering at night <is a sign> of the thief (indeed fancy dressing follows the adulterer, and wandering at night <follows> the thief), for this reason the rhetorical demonstrations are produced from the consequences. In order to show that <someone> is an adulterer, one takes the consequence that he dresses fancily, and in order to prove <that someone is> a thief <one takes the consequence> that he wanders at night. The confutation <of this argument> is that 'many have these <characteristics> but not the predicate'. <Aristotle> calls 'predicate' <1> adultery <qua> attribute of fancy dressing, and <2> thief <qua attribute> of wandering at night, not in the sense that they are true predicates (<he> has said indeed

is posterior to what is prior. But not all inferences from the posterior to the prior are inferences from effect to the cause, and thus sign-inferences. Morrison's straightforward equation of the two ('Philoponus' account counts any form of reasoning from posteriors to priors, or from effect to cause, as a tekmeriodic proof', 1997:10) is an unduly conflation of two things that are kept distinct by Philoponus, and finds no justification in the text. As to the second alleged difference between Philoponus and Aristotle, we have seen above that at 49, 5–14 Philoponus clearly distinguishes the syllogism from an irrefutable sign (ἄλυτα τεκμήρια, 49, 5–6) from the syllogism from a refutable sign (λυτόν σημεῖον, 49, 9–10). Only the former merits the title of demonstration, even though in a secondary sense: the former is a τεκμηριώδης ἀπόδειξις, while the latter is an inference from a σημεῖον, which being deductively invalid is no demonstration at all.

¹ Ebbesen (1981 I: 268–285) has shown that the text published by Wallies as [Alexander] *In SE* was in fact produced by Michel of Ephesus (12th cent.) on the basis of older sources; cf. also Ebbesen (1979: viii–xii).

that they do not necessarily convert), but in the sense that they are taken as predicates by the sophists. (our transl.)

An inference from the consequent to its antecedent is deductively invalid. Michael observes that according to Aristotle's explanation the proof of deductive invalidity is that though one has the one characteristic (the sign: fancy dressing, wandering at night), one will not necessarily have the other (the cause of the sign: being an adulterer, being a thief), that is to say, the effect has a wider extension than its alleged cause (and thus we are under sub-case II.b). Actually in Aristotle's example both fancy dressing and wandering at night are signs (in the strict sense) of being an adulterer, while according to Michael only the former is a sign of adultery, while the latter is a sign of theft. In either case, an inference from a consequent (the refutable sign) to one of its possible antecedents is a 'semiotic' (in the strict sense) demonstration, but not (what Philoponus calls) a 'tekmeriodic' one.

On the basis of the terminological parallel between Philoponus' τεκμηριώδης ἀπόδειξις and Michael's σημειώδης ἀπόδειξις one could be tempted to conjecture that Michael's source was Philoponus himself. The fact that in his commentary on the *Posterior Analytics* Philoponus explicitly denies the status of ἀπόδειξις to refutable sign-syllogisms is no real argument against the identification, as it may well be supposed that a refutable sign-syllogism is no ἀπόδειξις in the context of the theory of demonstrative science (the *Posterior Analytics*), while it can be called an ἀπόδειξις (though only a 'semiotic' one) in the context of the theory of rhetorical argumentation.

However that may be, Philoponus' terminological manoeuvre makes good sense indeed: while, as we have shown in §2 and §3, both Alexander and Themistius bring to the fore the contrast between sign and demonstration which was only hinted at in Aristotle's *Posterior Analytics*, it is Philoponus who uses the terminology of *APr*. II.27 in order to distinguish between deductively valid, i.e., 'tekmeriodic' demonstrations, and deductively invalid sign-inferences which do not deserve the dignity of demonstration at all. With his theory of 'tekmeriodic proof' Philoponus gives to Aristotle's scattered remarks on the relationship between sign and demonstration a precise terminological and conceptual systematization, which will be mostly completely ignored by subsequent commentators on the *Posterior Analytics*.¹

References

Allen, J. 2001. *Inference from Signs. Ancient Debates about the Nature of Evidence*. Oxford: Clarendon Press.

Barnes, J. 1969. Aristotle's Theory of Demonstration. *Phronesis* 14 (2): 123–152.

Barnes, J. 1981. Proof and the Syllogism. In Berti (ed.) *Aristotle on Science. The Posterior Analytics*. Padova: Antenore, 17–60.

Barnes, J. 1993. Aristotle. Posterior Analytics. 2nd edition. Oxford: Clarendon.

Barnes et al. 1991. *Alexander of Aphrodisias. On Aristotle Prior Analytics 1.1-7.* London: Bloomsbury.

Bellucci, F. 2018. Signs and demonstration in Aristotle. *British Journal* for the History of Philosophy 26 (5). DOI: 10.1080/09608788.2017.1420626

Borgo, M. 2009. Themistius on Demonstrative Premisses: A Reading of His Paraphrase of *Posterior Analytics*, 71b9-72a7. *Documenti e studi sulla tradizione filosofica medievale* 20: 149-192.

Burnyeat, M. F. 1981. Aristotle on Understanding Knowledge. In Berti (ed.) *Aristotle on Science. The Posterior Analytics*. Padova: Antenore, 97–139.

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Burnyeat, M. F. 1982. The origins of non-deductive inference. In Barnes et al. (eds.) *Science and Speculation*. Cambridge: Cambridge University Press, 193–238.

Goldin, O. 2009. Introduction. In *Philoponus (?). On Aristotle Posterior Analytics 2*. London: Bloomsbury, 1–10.

Gili, L. 2011. La sillogistica di Alessandro di Afrodisia. Sillogistica assertoria e sillogistica modale nel commento agli Analitici Primi di Aristotele. Hildesheim: Olms.

Ebbesen, S. 1979. Anonymi Aurelianensis I Commentarium in Sophisticos Elenchos. CIMAGL 34, University of Copenhagen.

Ebbesen, S. 1981. Commentators and Commentaries on Aristotle's Sophistici Elenchi. A Study of Post-Aristotelian Ancient and Medieval Writings on Fallacies, vols 1-3. Leiden: Brill.

Ebbesen, S. 2012. Review of *Interpreting Aristotle's Posterior Analytics in Late Antiquity and Beyond*, edited by. F. de Haas, M. Leunissen, M. Martijn (Leiden: Brill, 2010). *Aestimatio* 9: 355–366.

Manetti, G. 1993. *Theories of the Sign in Classical Antiquity*. Bloomington: Indiana University Press.

McKirahan, R. 1992. Principles and Proofs. Aristotle's Theory of Demonstrative Science. Princeton: Princeton University Press.

McKirahan, R. 2008. Introduction. In *Philoponus: On Aristotle Posterior Analytics 1.1–8*, London: Bloomsbury, 1–8.

Mignucci, M. 1975. *L'argomentazione dimostrativa in Aristotele. Commento agli Analitici Secondi*. Padova: Antenore.

Moraux, P. 1979. Le Commentaire d'Alexandre d'Aphrodise aux «Seconds Analytiques» d'Aristote. Berlin: De Gruyter, 1979.

Morrison, D. 1997. Philoponus and Simplicius on Tekmeriodic Proof. In Kessler (ed.), *Method and Order in Renaissance Philosophy of Nature:*

The Aristotle Commentary Tradition. Aldershot, Hampshire: Ashgate, 1–22.

Ross, D. 1949. Aristotle's Prior and Posterior Analytics. Oxford: Clarendon.

Smith, R. 1989. Aristotle. Prior Analytics. Indianapolis: Hackett.

Solmsen, F. 1929. *Die Entwicklung der aristotelische Logik und Rhetorik*. Berlin: Weidmann.